

## ONE-DIMENSIONAL STEADY FLOW-SEDIMENT MATHEMATIC MODELS HELIUS AND THEIR APPLICATION IN THE THREE GORGES PROJECT

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### ABSTRACT

In order to investigate the sediment problem of the Three Gorges Project (TGP), two 1-D steady flow-sediment mathematic models (i.e., HELIU-1 and HELIU-2) are developed gradually by Changjiang River Scientific Research Institute (CRSRI). Through calculation and verification for in situ measured data of large and medium scale reservoirs and channel reaches downstream of the dams in the Changjiang River Basin, it is found that they can give reasonable results to simulate reservoirs sedimentation and the downstream channel reaches' deformation due to scour in long time period and long distance. Especially, after completion of TGP the models are also used to predict reservoir sedimentation in the TGP Reservoir (780 km long) and the channel reach (over 1100 km long) downstream of the TGP Dam. The calculated results have been referred by the technical design department of TGP.

### 1. INTRODUCTION

At the beginning of the 1970s, a 1-D steady flow-sediment mathematic model HELIU-1 was developed for studying the sediment problem of the Three Gorges Project (TGP). This model was mainly used to study the scour and deposition of suspended sediment. It simulate reservoir sedimentation well, but it falls short of modeling the scour of river channel. Therefore, the model HELIU-2 was developed in the early 1990s. For HELIU-2, technical problems such as exchange of bed sediment and suspended sediment, river bed deformation and bed sediment composition have been taken into account.

This article introduces HELIU-1 and HELIU-2 and their application in three parts: the mathematic model, model verification and case studies. In the section of the mathematic model, the equations of HELIU-2 are introduced only, for HELIU-1 was introduced in the First International Symposium on River and Sedimentation, which was held in Beijing, China, in the early 1980s. In the section of model verification, the paper presents several sets of in situ measured data to validate the two models through many engineering examples. In the section of case studies, the results about the TGP Reservoir sedimentation and the downstream channel scour at the time when the TGP Reservoir will be operated for 100 years, are illustrated.

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## 2. HELIU-2 MATHEMATIC MODEL

### 2.1 Governing equations

According to the basic equations such as flow continuity equation, sediment continuity equation, sediment-carrying capacity equation and river bed deformation equation, and transforming them into finite difference equations, the following equations can be obtained<sup>[1, 3]</sup>.

(1) Water surface equation is

$$Z = Z_0 + \frac{n^2 Q^2 \Delta X}{2} \left( \frac{B^{4/3}}{A^{10/3}} + \frac{B_0^{4/3}}{A_0^{10/3}} \right) + \frac{U_0^2 - U^2}{2g} \quad (1)$$

(2) Variation equation of suspended sediment concentration is

$$S_i = S_{*i} + (S_{oi} - S_{*oi})e^{-Y} + (S_{*oi} - S_{*i})Y^{-1}(1 - e^{-Y}) \quad (i = 1, 2, \dots, 8) \quad (2)$$

in which:

$$Y = \frac{\alpha \omega_i \Delta x}{q}$$

$$S_{*i} = K_i S_{*m}$$

$$S_{*m} = k \left( \frac{U^3}{gh \omega_m} \right)^m$$

When  $m=0.92, k/g^m=0.0175$ ,

$$S_{*m} = 0.0175 \frac{Q^{2.76} B^{0.92}}{A^{3.68} \omega_m^{0.92}}$$

$$\omega_m^{0.92} = \sum_{i=1}^8 P_i \omega_i^{0.92}$$

$K_i$  is the coefficient of grouping sediment-carrying capacity, using Dou Guoren's Formula:

$$K_i = \frac{(P_i / \omega_i)^\beta}{\sum_{i=1}^8 (P_i / \omega_i)^\beta}$$

$P_i$  is the grading of suspended sediment, which is obtained from the following formula:

$$P_i = \begin{cases} P_{oi} & \text{equilibrium} \\ \frac{G_{soi} - \Delta G_{si}}{\sum (G_{soi} - \Delta G_{si})} & \text{non - equilibrium} \end{cases}$$

(3) River bed deformation caused by movement of suspended sediment is

$$\Delta Z_1 = \sum_{i=1}^8 \frac{(Q_0 S_{0i} - Q S_i) \Delta t}{\gamma_{st} B \Delta x} \quad (3)$$

(4) Bed load rate

It is obtained from the empirical curve presented by the CRSRI(Fig.1), and the corresponding formula is as follows:

$$\frac{V_d}{\sqrt{gd}} \sim \frac{q_s}{d\sqrt{gd}} \quad (4)$$

in which:

$$U_d = \frac{m+1}{m} \left(\frac{H}{d_i}\right)^{-\frac{1}{m}} U$$

$$m = 4.7 \left(\frac{H}{d_{50}}\right)^{0.06}$$

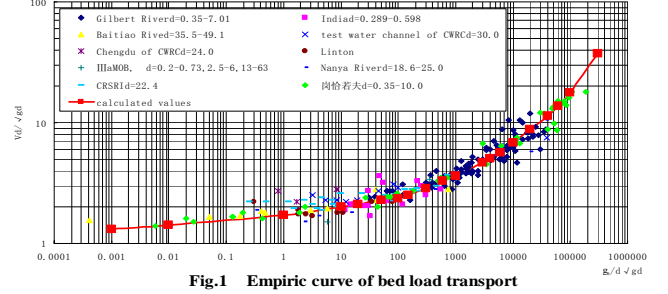


Fig.1 Empiric curve of bed load transport

(5) Sediment incipient velocity formula (Zhang Ruijin's Formula)

$$U_c = \left(\frac{h}{d}\right)^{0.14} \sqrt{17.6 \frac{\rho_s - \rho}{\rho} d + 0.000000605 \frac{10 + h}{d^{0.72}}} \quad (5)$$

(6) River bed deformation due to movement of bed load

$$\Delta Z_2 = \sum_{i=9}^{16} \frac{(G_{boi} - G_{bi}) \Delta t}{\gamma_{st} B \Delta x} \quad (6)$$

(7) Total deformation of the river bed

$$\Delta Z = \Delta Z_1 + \Delta Z_2$$

In the above-mentioned equations,  $\Delta t$  is the period of time;  $\Delta x$  is the distance between two sections;  $S_1, S_{*1}$  are the sediment concentration and sediment-carrying capacity respectively;  $S_{*m}$  is the total sediment-carrying capacity through the section;  $q$  is the unit discharge;  $\omega_m$  is the average settling velocity of nonuniform sediment;  $k, m$  are the coefficient and index of sediment-carrying capacity respectively;  $\beta$  is an index, adopted as  $1/6$ ;  $U_d$  is the flow velocity near the bed;  $U_c$  is the incipient velocity of bed sediment;  $d$  is the sediment grain size;  $H$  is the average water depth;  $g_b$  is the unit bed sediment transportation rate;  $G_b$  is the total bed sediment transportation rate; the footnote "0" represents the given section.

## 2.2 Solution of the coupled equations and treatment of related problems

In solving the simultaneous equations non-coupled solutions should be adopted. Firstly, to ascertain the water surface curve, and calculate the hydraulic elements; secondly, to find out the scour or deposition volumes for different reaches and all groups of sediment (including bed load sediment and suspended sediment); thirdly, to calculate the bed deformation (including modification of section configuration and bed sediment composition)

(1) Modification of section configuration. The section's characteristic curve is treated with the following ways: 1) If sediment deposits in a reach, its deposition along the wetted perimeter is supposed to be with the same depth; if scour occurs, it is supposed that the scour takes place only in the river bed, not in the river bank, which is generally for wide and shallow sections. 2) River bed is considered as a level due to scour or deposition, which is generally for narrow and deep sections.

(2) Treatment of wide sections. A wide section consists mainly of floodplains and a main channel. When a river has a medium or low water level, there will be no overflow on its floodplains. When it has a high water level, the mainstream will go through the main channel; in this case, the floodplains on which sediment would deposit will have a low water depth. Therefore, in calculating

the bed deformation the wide section will be divided into two parts (i.e., floodplains and a main channel) to treat simply.

### 3. VERIFICATION OF 1-D STEADY FLOW-SEDIMENT MATHEMATIC MODEL

#### 3.1 Verification of mathematic model HELIU-1

According to measured data from three reservoirs: Danjiangkou, Gezhouba and TGP, verification has been carried out for HELIU-1, the results are shown in Table 1. They show this model has a good simulation, and the errors compared with the measured data are reasonable.

In model verification on Gezhouba Reservoir and Danjiangkou reservoir [2], the coefficient  $k$  of the sediment-carrying capacity adopted was 0.03, the index  $m$  was 0.92, and recovery saturation

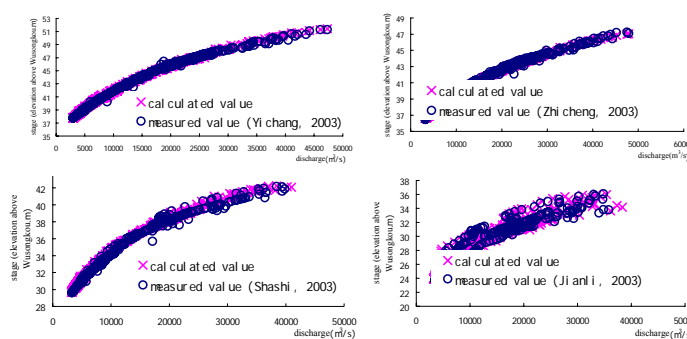
**Table1 Comparison of calculated and in situ measured sedimentation values for the large and medium scale reservoirs of the Changjiang**

River	Reservoir name	Reservoir length(km)	Operation period	Deposit volume			Deposit weight		
				Measured value ( $10^6\text{m}^3$ )	Calculated value ( $10^6\text{m}^3$ )	Error(%)	Measured value ( $10^6\text{t}$ )	Calculated value ( $10^6\text{t}$ )	Error(%)
Hanjiang	Danjiang	207	1968 - 1985	887.0	680.4	23.3	867	802	7.5
Changjiang	Gezhouba	162	1981 - 1985	145.1	160.3	-10.5			
Changjiang	TGP	760	Jun.2003—Oct. 2004				146	151	3.1

coefficient were 1.0 for scour state and 0.25 for deposition state. In verification of the TGP Reservoir's sedimentation after its first storage, the coefficient  $k$  were 0.02 for the reach upstream of Fuling and 0.03 for that downstream of Fuling, the index  $m$  was still 0.92, and recovery saturation coefficient were 1.0 for scour state and 0.25 for deposition state.

#### 3.2 Verification of mathematic model HELIU-2

The verification on HELIU-2 was done using the measured data (1980-1987) from the Yichang—Datong reach in the middle and lower Changjiang River, in which a large scour has occurred after completion of the Gezhouba Project. Its content included scour/deposition volumes and stage-discharge relations in main water level stations along the reach. Recently, the same verification has also been done using the measured data (June 2003-July 2004) from the Yichang—Chenglingji reach after the TGP Reservoir began to impound water. The verified results are shown in Table 2, Fig.2.



**Fig.2 Comparison of calculated and measured stage-discharge relations**

**Table2 Comparison of calculated and in situ measured scour/deposition volumes for the river reaches from Yichang to Datong**

Reach	Distance (km)	Jun.1980—Dec.1987			Oct.2002—Jul.2004		
		Measured value( $10^6\text{m}^3$ )	Calculated value( $10^6\text{m}^3$ )	Error(%)	Measured value( $10^6\text{m}^3$ )	Calculated value( $10^6\text{m}^3$ )	Error(%)
Yichang--Zhicheng	59	-43.48	-47.65	-9.6			
Zhicheng--Shashi	89	-34.27	-35.99	-5.0	-44	-54.18	-23.1
Shashi--Xinchang	67	-17.91	-17.47	2.5	-36.97	-34.12	7.7
Xinchang--Jianli	91	-6.65	-9.21	-38.5	-35.08	-34.67	1.2
Jianli--Chenglingji	92	-50.11	-48.48	3.3	-29.61	-29.01	2.0
Yichang--Chenglingji	398	-152.42	-158.8	-4.2			
Zhicheng--Chenglingji	339				-116.05	-122.97	-6.0
Chenglingji--Wuhan	230	175.1	185.2	-5.8			
Wuhan--Datong	500	-20	-17	15.0			

note: "+" means deposition, "-" means scour.



it will keep 85% of the designed flood control storage and 91% of the designed regulating storage, which will make it continuously operate for a very long time.

**Table3 Distribution of the reservoir's sedimentation in 100 years after its first storage**

Reach	Zhutuo--Chongqing	Chongqing--Fuling	Fuling--damsite	Region of its trunk stream
Distance	148km	121km	491km	760km
Deposition ( $10^9 m^3$ )	in the end of 30 years	0.038	0.391	8.143
	in the end of 50 years	0.055	0.578	12.240
	in the end of 80 years	0.094	0.858	14.767
	in the end of 100 years	0.133	0.954	15.565

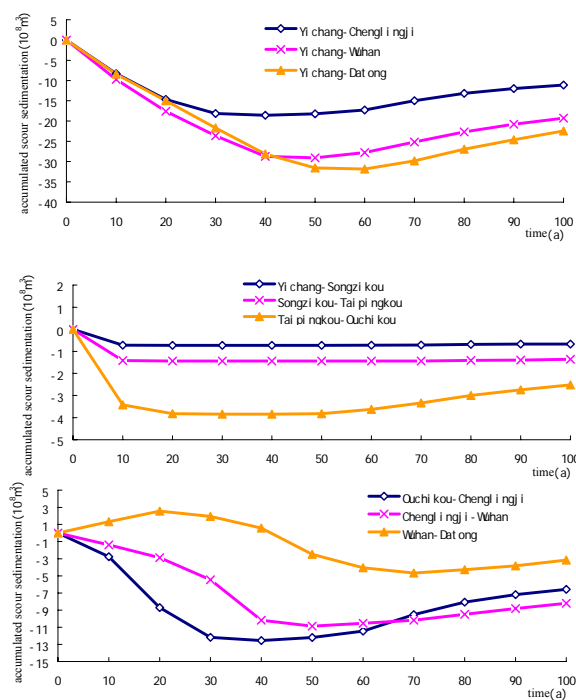
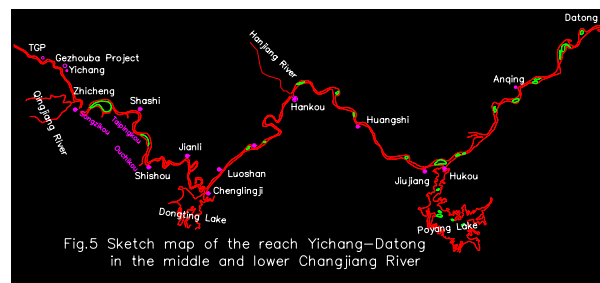
## 4.2 Application of HELIU-2 in predicting the influence of the TGP Dam on scour/deposition in its downstream reach

The model HELIU-2 emphasizes to study the problem to forecast the scour/deposition variation trend of a large or medium scale reservoir and its downstream reach after the flow-sediment conditions have been changed due to building the dam. It has been used to forecast the 100-year scour/deposition variation of the reach Yichang—Datong (over 1100km long, see Fig.5) after the first impoundment of the TGP Reservoir.

Selecting the calculated 100-year outflow sediment processes after operation of the TGP Reservoir as the reach's inflow sediment condition and adding the afflux of its upstream tributaries and lakes, which were obtained from the 10-year series (1961-1970) data, the influences of building the TGP on the middle and lower reaches of the Changjiang River were predicted, including scour/deposition, processes, bed load coarsening and low water level dropping along the reach.

### (1) The predicted scouring amount and scouring processes

During the early period of reservoir impoundment, a lot of deposit will occur in the reservoir, and the sediment discharged out from it will decrease markedly, the grain size of which will also become finer. Here the discharged flow will have a larger scour effect on the downstream reaches. With the increase of time, sedimentation in the reservoir will decrease gradually, reaching the balance of scour and deposition; and the sediment discharged downstream will increase, and then the back deposit will take place in the downstream reach. The processes of Scour—Balance—Back-deposit are the variation law of the downstream reach, and also is the developing trend of scour of the reach Yichang-Datong (over 1100km long). Fig.6 shows the processes. The maximum cumulated scour volume of the whole reach is 3.18 billion  $m^3$ , appearing at the sixtieth year. The scouring rate is the largest in the first 10 years, 0.1



**Fig.6 Accumulated scour sedimentation processes of the reaches from Yichang to Datong**

billion  $m^3$  per year on an average. From then on, it will gradually become smaller till the maximum value produces at the sixtieth year, then the back deposit will appear in the whole reach.

Through the forecast, the maximum cumulated scour volume in the reach Yichang-Chenglingji will be 1.855 billion  $m^3$ , presenting at the fortieth year after impoundment of the TGP Reservoir. In the reach Chenglingji-Wuhan it will be 1.088 billion  $m^3$ , presenting at the sixtieth year. In the reach Wuhan-Datong it will be 0.466 billion  $m^3$ , presenting at the seventieth year. By the time that the maximum cumulated scour volumes occur, the back deposit will begin in the reaches, and the scour/deposition processes in these reaches agree with the law of Scour—Balance—Back-deposit basically.

### (2) Coarsening of river bed due to scour

In the scour processes the downstream reach is eroded by the ways “depositing the coarse and suspending the fine” and “scouring the fine and remaining the coarse”. The coarser sediment from the upper reach will exchange with the finer sediment in the calculated reach, coarsening the bed composition. When this mechanism of exchanging scour occurs repeatedly along the channel, the bed sediment median diameter  $d_{50}$  will become smaller (Fig. 7), while the  $d_{50}$  in the calculated section will become larger with the increase of time (Fig. 8).

After completion of the TGP, the downstream scour will accelerate, and the gravel in the channel will further transmit downstream. The calculation forecasts that in 30 years after operation of the TGP Reservoir the gravel with its  $d_{50}$  more than 10mm will approach to Xinchang, 220km away from Yichang. The  $d_{50}$  gravel more than 1.0mm will be near Shishou, 250km from Yichang. In other words, in 30 years the bed surface layer of the upper Jingjiang River (Zhicheng--Ouchikou) will be covered with gravel or gravel with sediment.

### (3) Lowering of the low water level due to scour

After operation of the reservoir the scour conditions of various downstream reaches have large differences in time and space due to variation of discharged sediment amount from the reservoir and different bed compositions. With the increase of time, the stage-discharge relations at various stations in the reach Yichang—Wuhan will change and the stages will appear to be lowered gradually. In the end of 30 years after operation of the reservoir, if the discharge is  $5500m^3/s$  at Yichang station the stage there will have a decrease of 0.95m than that in 1993, 2.07m lower than the designed stage in 1973 (Fig.9); then the stage at Shashi will be 2m less than in 1993 (Fig.10); the maximum stage at Shishou will lower by 3.3m. In 50 years after operation of it, the scour in the

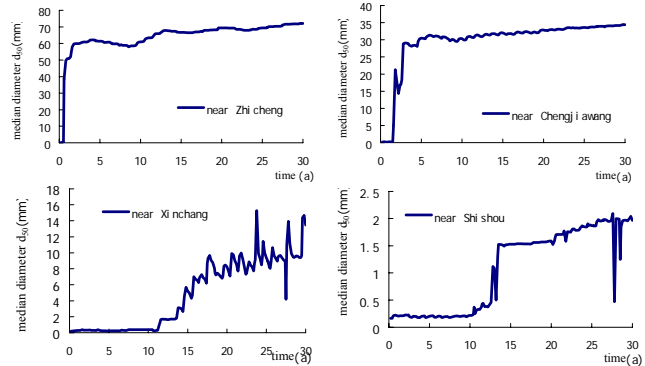


Fig.8 Variation processes of  $d_{50}$  of bed sediment of typical sections

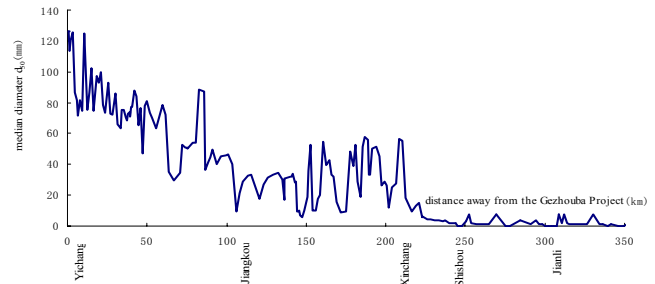


Fig.7 Variation process of  $d_{50}$  of bed sediment of the dam-downstream reach in the end of 40 years after the reservoir's first storage

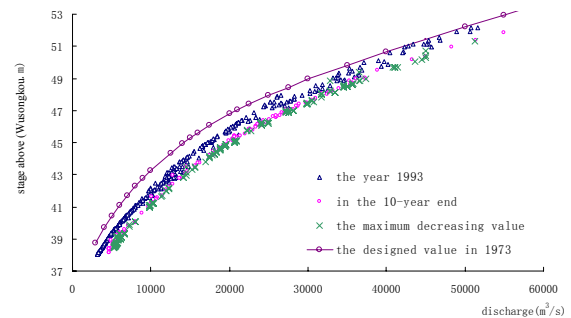


Fig.9 Lower trend of the stage at Yichang station

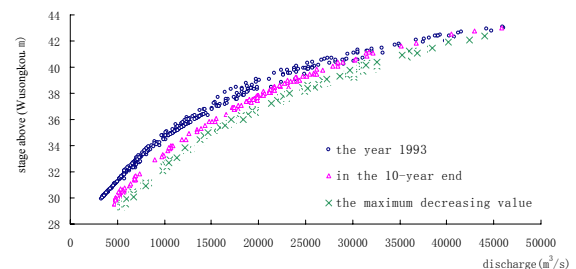


Fig.10 Lower trend of the stage at Shashi station

reach Chenglingji—Wuhan will cease, the average eroded depth will be 2.49m, and the lower stage at Luoshan station will decrease by 1.87m. In 60 years the scour in the reach Wuhan—Jiujiang will basically stop, the average eroded depth will be 0.6m and the lower stage at Wuhan station will decrease by 0.77m.

After operation of the TGP Reservoir, the long-distance and long-time scour will occur in the middle and lower reaches of the Changjiang River, and the medium and lower water levels will decrease obviously; at the time of reaching the scour equilibrium, the dam-downstream reaches will begin to deposit back step by step from the top down, and the stages along the channel will cease decreasing or even will have a little rising.

## 5. CONCLUSIONS

(1) This paper uses the practical data after completion of the TGP Reservoir to verify the models HELIU-1 and HELIU-2. The obtained results have shown that these two models can fairly well simulate the sedimentation law of reservoir and the scour law of downstream river channel. And they have been put in use in planning and design large and medium-scale reservoirs.

(2) The forecasted results given in the paper, which are about reservoir sedimentation and the long-time period and long-distance scour of the downstream channel, have been certificated and cited by the engineering design department.

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