

APPLICATION OF TWO-DIMENSIONAL MATHEMATICAL MODEL TO THE STUDY OF THE YANGTZE RIVER WATERWAY IMPROVEMENT

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ABSTRACT

Based on the deep analysis of the change characteristic of numerous shallow and dangerous waterways in the middle and lower reaches of the Yangtze River, a two-dimensional mathematical model is established by using the finite volume method. A general empiric formula for the bed resistance coefficient is inferred from amounts of actual measurement data. The modeling is verified in Lujiahe, Wuxue and Taiziji waterway. The calculated results indicate that the modeling fully reflects the current movement behavior of the middle and lower reaches of the Yangtze River. Lujiahe waterway is one of the shallow and dangerous waterways that are earlier and highly affected by the Three Gorges project. From the experiment of modeling, this paper predicts the influence of the operation of the Three Gorges Reservoir on this waterway. In fact, Wuxue waterway is the only restricted one-way navigational shoal in the middle reaches of the Yangtze River. By using the modeling, this paper makes a deep study for Wuxue waterway. Taiziji waterway is a typical braided reach of goose-head-type. On the basis of the numerical simulation, this paper compares and demonstrates multiple schemes, and finally puts forward a proper reef-blasting one, which provides a scientific base for the improvement of Taiziji waterway.

1. INTRODUCTION

The Yangtze River, known as a “golden waterway”, is the largest river in China. Up to now, apart from a few shoals of the middle reaches in the Yangtze River, which have been improved, most waterways are in natural conditions. Over 20 years of the opening and reform to the outside world, Chinese economy has developed rapidly. The present situations of the Yangtze River hardly meet the requirement and development of the water transport industry. In order to solve this problem, since the outset of the 21st century, the large-scale research and improvement of the middle reaches of the Yangtze River has begun and the mathematical model, as an important means of research, has been widely used and played an important role in the research on the Yangtze River waterway improvement.

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On the basis of the deep analysis of the change characteristic of numerous shallow and dangerous waterways in the middle and lower reaches of the Yangtze River, a two-dimensional mathematical model is established by using the finite volume method. A general empiric formula for the bed resistance coefficient is inferred from a great number of measured data. The results verified by many waterways indicate that the modeling fully reflects the flow movement behavior in the middle and lower reaches of the Yangtze River. Taking Lujiahe, Wuxue and Taiziji waterways as examples, this paper presents the application of the two-dimensional mathematical model to the study of the Yangtze River waterway improvement.

2. ESTABLISHMENT OF MODELING

2.1 Basic Equations

Continuity equation of flow:

$$\frac{\partial h}{\partial t} + \frac{\partial uh}{\partial x} + \frac{\partial vh}{\partial y} = 0 \quad (1)$$

Equation of flow movement:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + g \frac{\partial z}{\partial x} + g \frac{n^2 u \sqrt{u^2 + v^2}}{h^{4/3}} = \varepsilon \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + g \frac{\partial z}{\partial y} + g \frac{n^2 v \sqrt{u^2 + v^2}}{h^{4/3}} = \varepsilon \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (3)$$

Equation of suspended load transport:

$$\begin{aligned} & \frac{\partial h s_i}{\partial t} + \frac{\partial u h s_i}{\partial x} + \frac{\partial v h s_i}{\partial y} - \varepsilon_s \left[\frac{\partial}{\partial x} \left(h \frac{\partial s_i}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial s_i}{\partial y} \right) \right] \\ & = \alpha \omega_i (s_* - s) \end{aligned} \quad (4)$$

Equation of deformation of riverbed with suspended load:

$$\rho_s \frac{\partial \eta_{si}}{\partial t} + \frac{\partial q_{ssx}}{\partial x} + \frac{\partial q_{ssy}}{\partial y} = 0 \quad (5)$$

Equation of deformation of riverbed with bed-load:

$$\rho_b \frac{\partial \eta_b}{\partial t} + \frac{\partial q_{sbx}}{\partial x} + \frac{\partial q_{sby}}{\partial y} = 0 \quad (6)$$

Formula for sediment-carrying capacity of flow

$$s_* = s_*(\bar{u}, h, \omega, \dots) \quad (7)$$

Formula for transport rate of bed-load

$$\bar{q}_{sb} = \bar{q}_{sb}(\bar{u}, h, d, \dots) \quad (8)$$

Where h and \bar{u} stand for the vertical water depth and the average vertical velocity respectively, $\bar{u} = \sqrt{u^2 + v^2}$; u and v stand for the average vertical velocity component in the direction of x and y ; S and s^* stand for the average vertical sediment concentration and sediment-carrying capacity respectively; ε stands for the turbulent viscous coefficient; ε_s stands for the sediment diffusion coefficient; \bar{q}_{sb} stands for the transport rate of bed-load per unit width; q_{ssx} and q_{ssy} stand for the transport rate of suspended load per unit width in the direction of x and y respectively. q_{sbx} and q_{sby} stand for the transport rate of bed-load per unit width in the direction of x and y respectively.

2.2 Main Parameters

2.2.1 Determination of Roughness Coefficient

The resistance of a natural river channel is very complicated, which contains the resistance of sediment grain, the resistance of sand ripples, the resistance of channel planform and the resistance of shaping accretion in the river. In the middle and lower reaches of the Yangtze River, the channel is wide and deep, composed of sandy riverbed, therefore, as far as the calculation control point is concerned, the difference of roughness coefficient lies mainly in the resistance of the channel planform and the resistance of shaping accretion in the river. By analyzing the data measured in the Yangtze River, the empirical formula for the roughness coefficient in a horizontal plane is determined as follows:

$$n = C_1 \cdot (1 + (1/h)^{n_1}) \cdot f(A, r, u)$$

Where C_1 stands for the one-dimensional roughness coefficient, h stands for the water depth, A stands for the flow area, r stands for the bend radius, u stands for current velocity. With the difference of the channel planform, the formula $f(A, r, u)$ will be a little bit different and its value approximately ranges from 0.5 to 1.5.

2.2.2 Sediment-carrying Capacity of Flow

By analyzing the measured data, the empirical formula is determined as follows:

$$S_* = K \left(0.1 + 90 \frac{\omega}{\bar{u}} \right) \frac{\bar{u}^3}{gh\omega}$$

Where K stands for the average sediment-carrying capacity coefficient in section. The value to be selected for K is the same as one-dimensional sediment-carrying capacity coefficient. The sediment-carrying capacity in groups is determined according to document ^[1].

3. CASE STUDY

The established mathematical model was applied in the research on the regulation works for some waterways of the middle reaches of the Yangtze River such as Lujiahe, Jiepai, Lianziwan, Shashi, Majiazui, Wuxue and Taiziji waterways. Before the mathematical model was applied in the research on every waterway, the mathematical model had been verified by the measured data. The verified results indicate that the established model is reasonable and rational, fully reflecting the flow and sediment movement behavior in the middle and lower reaches of the Yangtze River.

Taking Lujiahe, Wuxue, Taiziji waterways as examples, this paper introduces the application of mathematical model to the understanding of the flow and sediment movement behavior, the channel change predication and the project justification.

3.1 Lujiahe Waterway

Lujiahe waterway is in the middle reaches of the Yangtze River, starting at Zhicheng and ending at Dabujie, totally 52 km long. This waterway is located in a transitional zone from the mountainous river to the plain river, which consists of Guanzhou, Lujiahe, Zhijiang, Liuxiang, Jiangkou and Dabujie waterways (see Fig. 1).

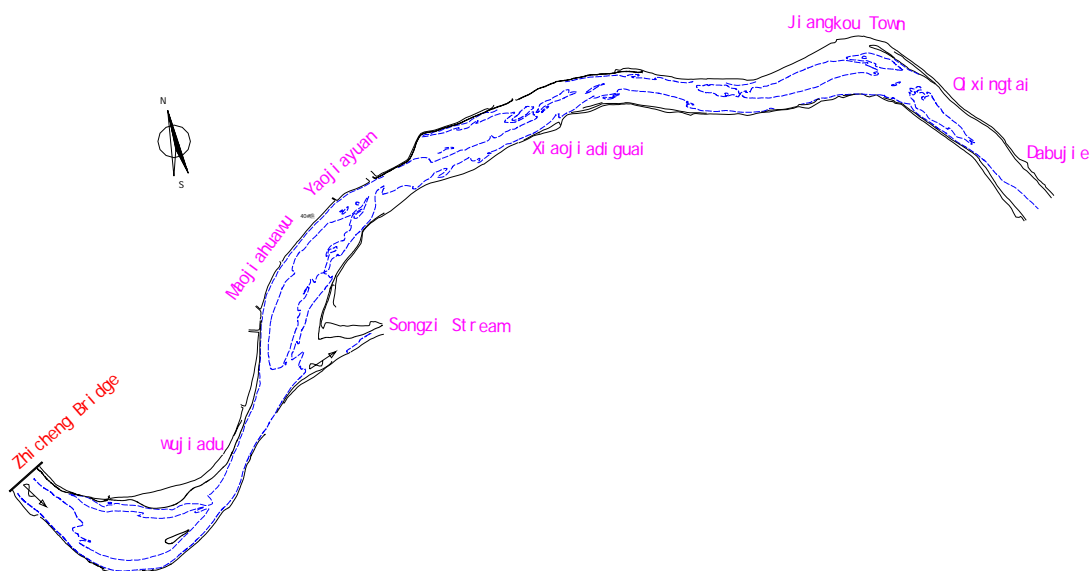


Fig. 1 Lujiahe Waterway

After operation of Three Gorges Reservoir, the sediment content in the water from Three Gorges Dam reduces substantially, bed erosion will widely exist in a long distance of river section downstream of Gezhouba dam, resulting in a new problem to the navigation channel. In the sand-pebble shoal section from Zhijiang to Dabujie, the erosion, roughening and stability of the riverbed will be completed in a short period of time. In case that the sandy riverbed just downstream will be eroded substantially, it will draw the people's attention whether the sand-pebble river section and the transient section from the sand-pebble river section to the sandy section will be deteriorated or not. The section from Zhijiang to Dabujie is of sand-pebble river bed, 76km downstream of Three Gorges Dam, one of the shallow and dangerous waterways that are earlier and highly affected by the Three Gorges Project. To have a correct understanding of the influence of Three Gorges Project on this section is essential for the waterway maintenance and the research on the improvement alternatives.

Based on the future 20 years' water-sediment data calculated by Changjiang Water Resources Commission, the model calculated the change of this section after operation of the Three Gorges Reservoir. The results are as follows:

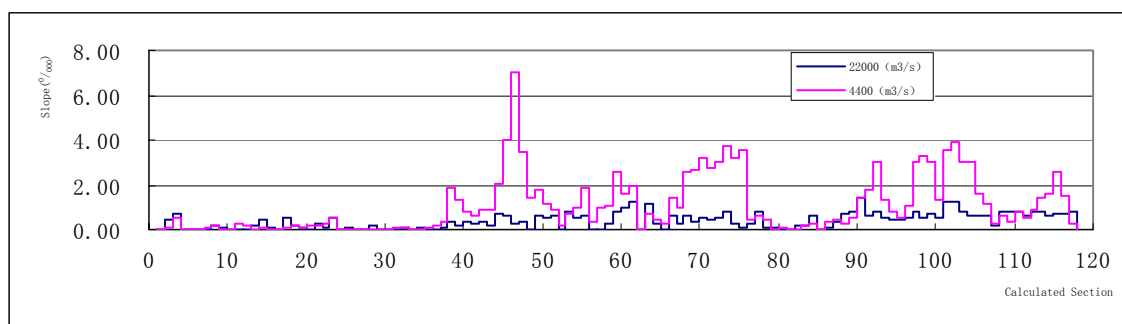


Fig. 2 Variation in the Slope along Lujiahe Waterway

a. After operation of the Three Gorges Reservoir, the channel erosion downstream of Gezhouba Dam widely exists. Lujiahe waterway has an obvious steep slope, the maximum being about $7^{0}/_{000}$ (see Fig.2).

b. During the first 20 years of operation of the Three Gorges Reservoir, the high current velocity areas in the dry season from Zhicheng to Dabujie mainly distribute in section between Daogujingoushi and 40# rock, in the crossing channel between Xiaojiatiguai and Zhijiang and in the crossing channel between Wujiadu and Qixingtai and the maximum velocity of flow is 2.68m/s, 3.21m/s and 3.17m/s respectively.

c. During the first 20 years of operation of the Three Gorges Reservoir, in the section between Zhicheng to Dabujie, the channel is over 3m deep in the dry season. But unfavorable navigation conditions such as narrow channel width, steep gradient and swift current occur in such areas as Majiahuawu, Zhijiang and Wujiadu.

d. After operation of the Three Gorges Reservoir, the diversion ratio of the left branch at Guanzhou is between 10% and 30% in the dry season. When the discharge of the Yangtze River is below $6000\text{m}^3/\text{s}$, there is no flow passing through Songzi River, a tributary of the Yangtze River. The diversion ratio of Songzi River will increase as the discharge of the Yangtze River increases, the maximum diversion ratio being about 10%.

The operation of the Three Gorges Reservoir began in June 2003. The recent observation data indicate that the bed scour widely occurs in the reach downstream of Gezhouba dam. In Lujiahe waterway, the phenomenon “steep slope and swift current” occurs, the maximum gradient occurs next to 40# rock and the measured maximum gradient is $7.75^{0}/_{000}$. The survey sheet in the dry season of 2004 shows that the water depth is more than 3m in Lujiahe Waterway in the dry season, but the unfavorable situations of steep slope and swift current have obviously occurred in Lujiahe waterway. The predicted result by the model is the same as the real one.

3.2 Wuxue Waterway

Wuxue waterway is located in the middle reaches of the Yangtze River, starting at Xiangushan and ending at Hulushan, 45km upstream of Jiujiang city of Jiang'xi province, with a total length of 14.5km(see Fig. 3). Under natural conditions, the river regime in Wuxue waterway is relatively stable, the bed form maintains the pattern of “two channels and one shoal”. The north channel is wide and shallow and the south channel is narrow and deep. Since 1930 the south channel has been the main channel in the dry season. Because the entrance of the south channel is narrow and

shallow, the south channel of Wuxue waterway is the only navigation channel restricted to one-way traffic in the middle and lower reaches of the Yangtze River.

The mathematical model focused on the calculation of the flow structure of Wuxue waterway. On the basis of calculation, the improvement scheme for this waterway was put forward and the engineering result of the improvement scheme was demonstrated.

The calculated result shows a long distance and a wide range of floodplain flow is the main characteristic of this waterway evolution. Fig. 4 and 5 show the velocity vector at different magnitudes of discharge. Seen from Figs. 4 and 5, the lower the water level is, the more floodplain flow concentrates on the head of Ya'erzhou central bar.

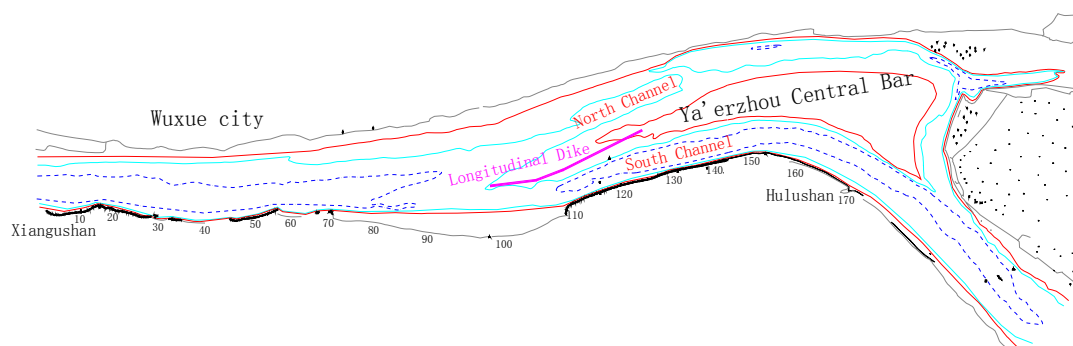


Fig.3 Wuxue waterway

Fig.6 shows that, under natural conditions, the maximum current velocity in the shallow area at the entrance of the south channel is between 0.8 m/s and 1.0m/s. The lower current velocity is one of the reasons why the entrance of the south channel is narrow and shallow. Therefore, to increase the current velocity at the entrance of the south channel and enhance the erosion of the shallow area is the basic measure to improve the navigation condition in the south channel.

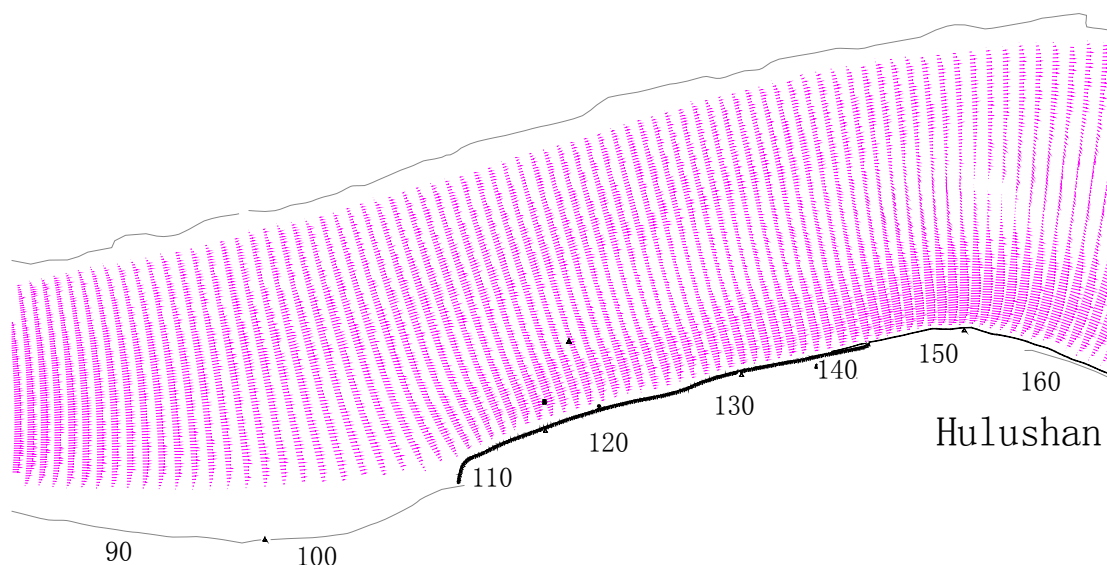


Fig.4 Local Flow Velocity Vector of Wuxue Waterway ($Q=16700\text{m}^3/\text{s}$)

The different floodplain flow structures at different water levels are mainly caused by the shape of Ya'erzhou central bar. To properly increase the height of the ridge of Ya'erzhou central bar is one of the important measures to increase the diversion ratio and the current velocity at the entrance of the south channel.

The mathematical model was used in the research on the calculation of eight improvement schemes. One improvement scheme was recommended that a longitudinal dike should be built on Ya'erzhou central bar after many factors had been taken into consideration such as the improvement result, the effect on Wuxue port and the utilization of the left bank line and levee.

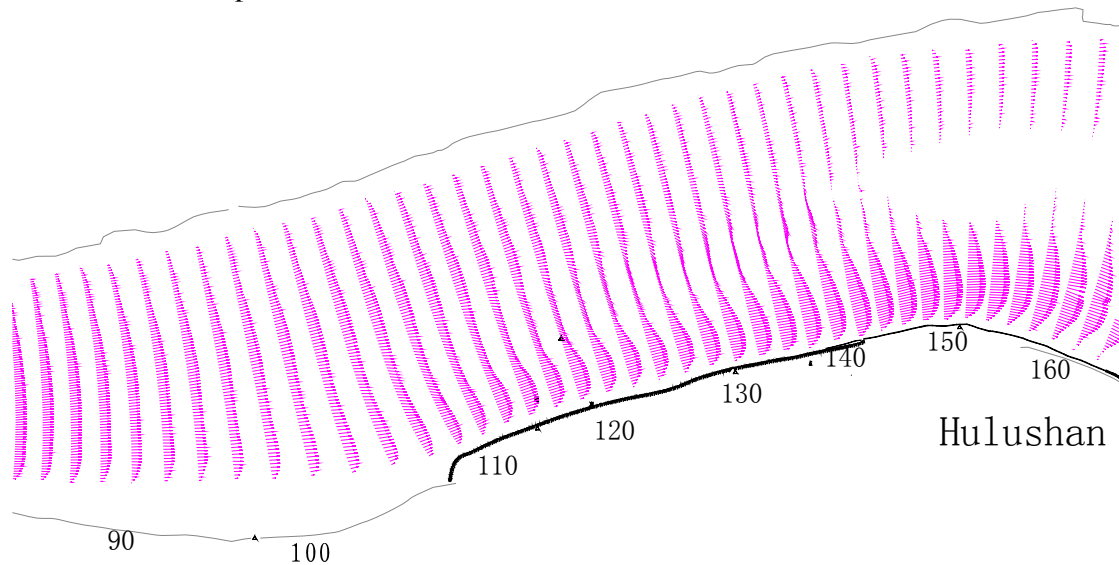


Fig.5 Local Flow Vector of Wuxue Waterway ($Q=10400\text{m}^3/\text{s}$)

The calculated result shows that, when the discharge is $16,700\text{m}^3/\text{s}$, the longitudinal dike is submerged and basically there is no change in the flow structure of Wuxue waterway before and after the project. When the discharge is $10,400\text{m}^3/\text{s}$, the longitudinal dike is not submerged, the floodplain flow on Ya'erzhou central bar is held back by the longitudinal dike and most water rounds the head of the longitudinal dike and enters the south channel. Before the project, the diversion ratio at the entrance of the south channel is 32.6%. After the project, the diversion rate at the entrance of the south channel is up to 48.5% and the current velocity in the shallow area ranges from 1.2m/s to 1.4m/s, increasing by about 60%.

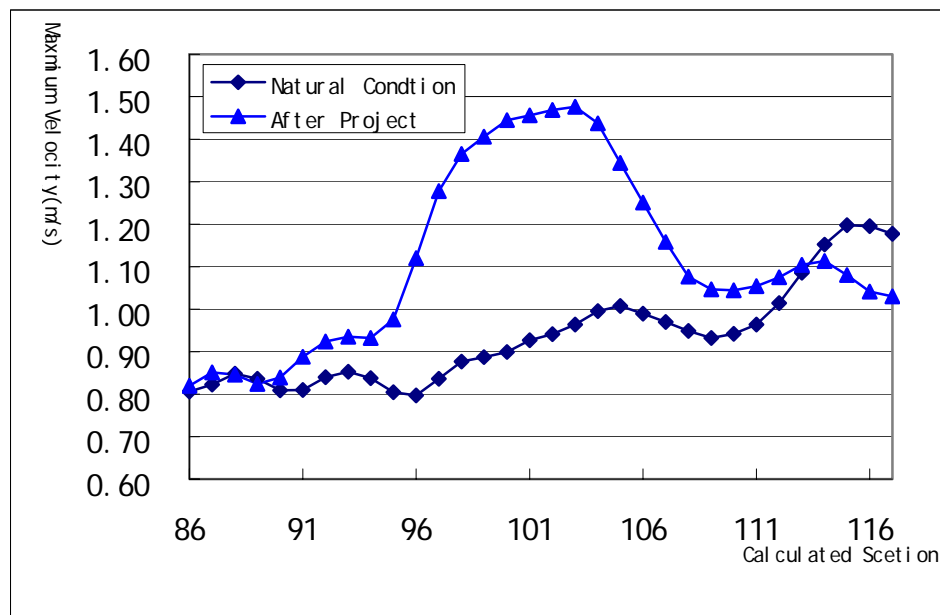


Fig.6 Variation in the Highest Current Velocity along the Entrance of the South Channel before and after the Project ($Q=10,400\text{m}^3/\text{s}$)

3.3 Taiziji Waterway

Taiziji waterway, located in the lower reaches of the Yangtze River, is 21km long from Qianjiangzui to Qiliji, one of the shallow and dangerous waterways affecting the navigation in the lower reaches of the Yangtze River (see Fig. 7). On the right bank of the upper section of this waterway exists a river cliff called Lanjiangji (see figure 8), standing out over 700m. The lower section is a braided section of goose-head type, with an island in it. The island divides the waterway into the North channel and the South channel. The South channel is sub-divided into Donggang and Xigang branches.

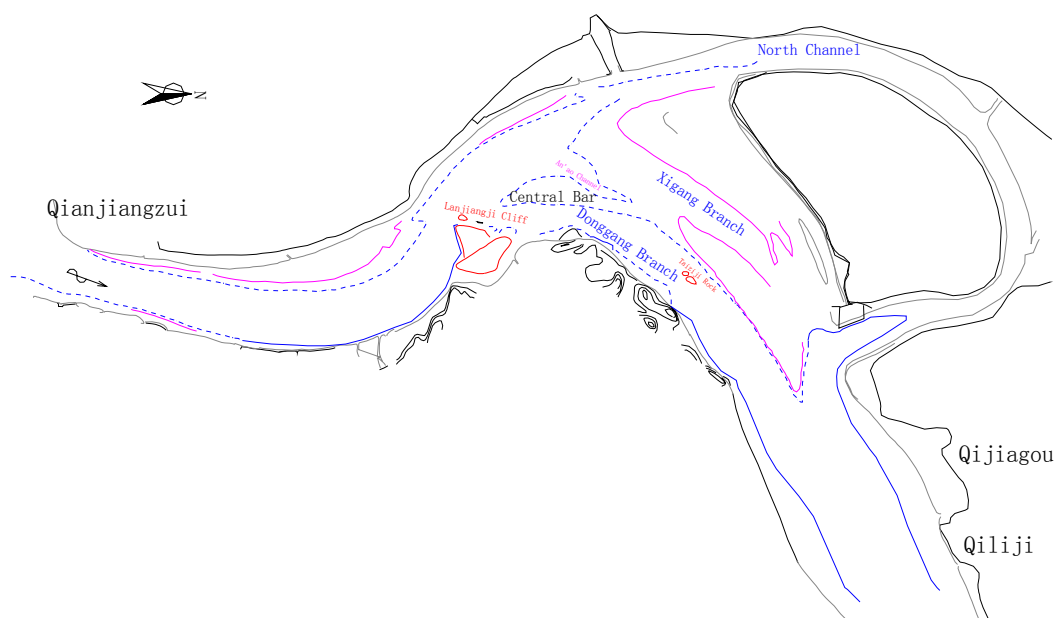


Fig.7 Taiziji waterway

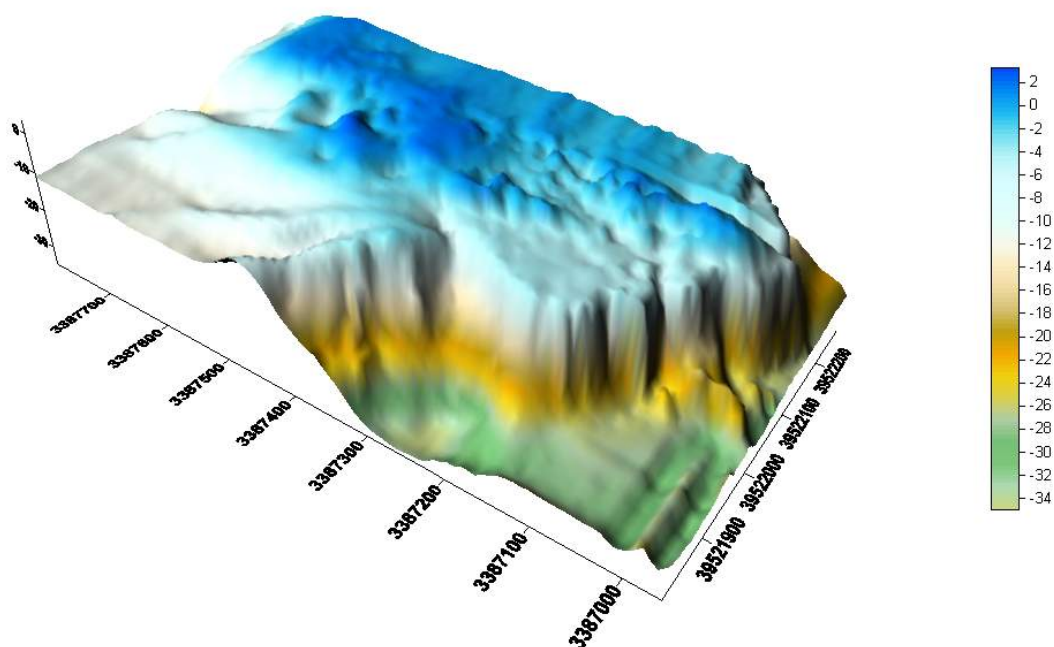


Fig.8 Three-dimensional Topographical Map of River Cliff

branch and Xigang branch. In the middle portion of Donggang branch, there is a huge rock (Taiziji rock) protruding above the water surface. In recent years, with the fast development of water transport, the traffic density increases rapidly. However, influenced by the river cliff and Taiziji rock, major marine accidents often took place, which made the transport sector suffer a great loss.

The Mathematical model calculated 21km of the channel from Qianjiangzui to Qijiagou. The 150×800 grid pattern was used. The average spacing between transverse grid lines is 10 to 20m and the average spacing between longitudinal grid lines is 25m. The transverse grid lines are basically perpendicular to the longitudinal grid lines.

The Mathematical model was used in the calculation of four improvement schemes, focusing on demonstrating the improvement of flow condition and the effect on the river channel on the premise of satisfying the channel dimensions. The arrangement of the schemes can be seen in table 1 and Fig. 9:

Table 1 Arrangement of Calculation Schemes for Taiziji waterway

Schemes	Arrangement	
	Lanjiangji(river cliff)	Taiziji rock
I	The portion to be cut is 110m long, the water depth is up to 5.0m after explosion and the volume to be removed is $188,900\text{m}^3$	The water depth is up to 5.0m after explosion and the volume to be removed is $80,900\text{m}^3$.
II	The portion to be cut is 180m long, the water depth is up to 5.0m after explosion and the volume to be removed is $287,100\text{m}^3$	The water depth is up to 5.0m after explosion and the volume to be removed is $80,900\text{m}^3$.
III	The portion to be cut is 230m long, the water depth is up to 5.0m after explosion and the volume to be removed is $395,300\text{m}^3$	The water depth is up to 5.0m after explosion and the volume to be removed is $80,900\text{m}^3$.
IV	The portion to be cut is 180m long, the water depth is up to 5.0m after explosion and the volume to be removed is $298,500\text{m}^3$	The water depth is up to 5.0m after explosion and the volume to be removed is $80,900\text{m}^3$.

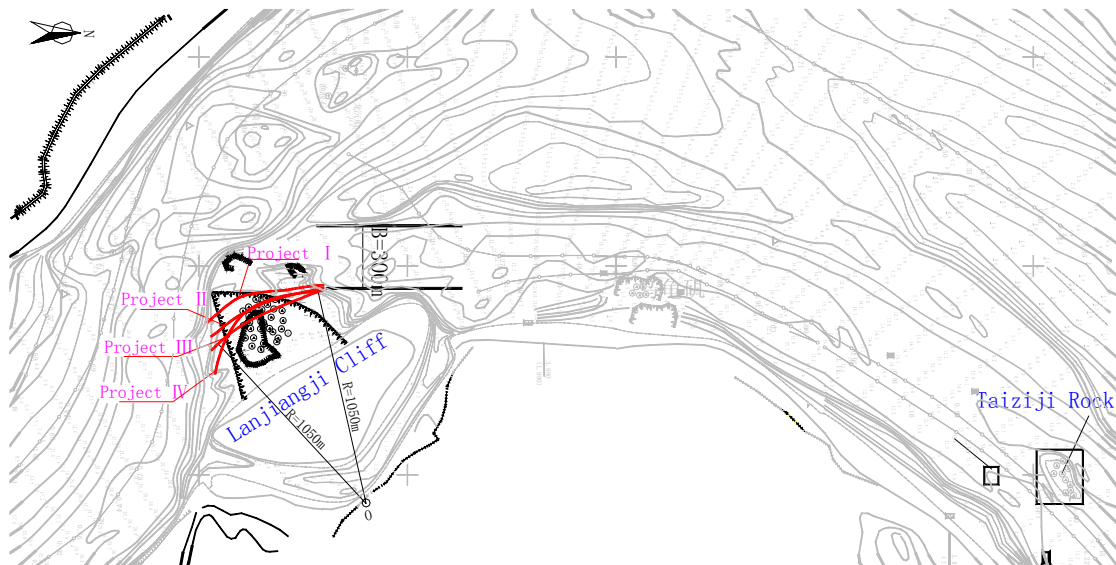


Fig.9 Arrangement of Calculation Schemes for Taiziji waterway

3.3.1 Improvement of the Navigation Conditions

Under natural conditions, the outside rock of Lanjiangji comes out of water in the dry season and there is a backflow area with a diameter of 150m near the entrance of Donggang branch, downstream of Lanjiangji. After scheme I is put into effect, the diameter of backflow area will reduce to about 120m, the center of backflow will move back about 20m to the rock than before the project. After scheme II is carried out, in the dry season the diameter of the backflow area near the entrance of Donggang branch will reduce to about 80m and the center of backflow moves back about 200m to the rock than before the project. When scheme III and scheme IV are carried out, the backflow will disappear.

Under natural conditions, due to the influence of the channel planform and Lanjiangji (river cliff), the included angle between the flow direction and the shipping line at the entrance of Donggang branch is relatively big, about 28° at most. After the implementation of the schemes, it is found that the oblique flow at the entrance is relatively smooth and the drift angle reduces. For scheme I, scheme II, scheme III and scheme IV, the drift angle reduces by $2-4^{\circ}$, by $4-7^{\circ}$, by $10-11^{\circ}$ and by $6-9^{\circ}$ respectively.

3.3.2 Effect of the Project on the River Channel

Tables 2 and table 3 list the diversion ratios of the different channels at different magnitudes of discharge before and after the project. From the tables, it is found that, after the project is carried out, there isn't much influence on the distribution of discharge in Taiziji waterway.

Table 2 Comparison of Diversion Ratios for Taiziji Waterway before and after the Project (%)
($Q=35,300\text{m}^3/\text{s}$)

	Before Project	Scheme I		Scheme II		Scheme III		Scheme IV	
		After Project	Increment	After Project	Increment	After Project	Increment	After Project	Increment
Entrance of Donggang Branch	28.06	28.64	0.58	29.23	1.17	30.11	2.05	29.46	1.4
Overbank Flow on the Central Bar	7.23	6.85	-0.38	6.5	-0.73	6.01	-1.22	6.37	-0.86
An'ao Channel	46.64	46.54	-0.1	46.34	-0.3	46.02	-0.62	46.28	-0.36
Xigang Branch	1.75	1.76	0.01	1.76	0.01	1.76	0.01	1.76	0.01
North Channel	16.32	16.21	-0.11	16.17	-0.15	16.1	-0.22	16.13	-0.19

Table 3 Comparison of Diversion Ratios for Taiziji Waterway before and after the Project(%) ($Q=12050\text{m}^3/\text{s}$)

	Before Project	Scheme I		Scheme II		Scheme III		Scheme IV	
		After Project	Increment	After Project	Increment	After Project	Increment	After Project	Increment
Entrance of Donggang Branch	34.3	35.03	0.73	34.88	0.58	34.77	0.47	34.8	0.5
Overbank Flow on the Central Bar	16.21	15.92	-0.29	15.96	-0.25	15.97	-0.24	16.01	-0.2
An'ao Channel	37.22	37.17	-0.05	37.25	0.03	37.34	0.12	37.36	0.14
Xigang Branch	0.14	0.14	0	0.14	0	0.13	-0.01	0.14	0
North Channel	12.13	11.74	-0.39	11.77	-0.36	11.79	-0.34	11.69	-0.44

3.3.3 Recommended Scheme

From the above calculated results, scheme II and scheme IV are superior to another two schemes in the improvement of navigation condition at the entrance of Donggang branch. Scheme II and scheme IV only differ in the plan layout of the portion to be removed and there is a little difference between the improvement results. The comparison of the two schemes is as follows:

From the table 4, both scheme II and scheme IV can greatly improve the navigation conditions of Taiziji waterway. By contrast, scheme IV is better than scheme II, therefore, scheme IV is selected as a recommended scheme.

4. CONCLUSIONS

The middle and lower reaches of the Yangtze River are located in the plain, with many economic, cultural, scientific and industrial towns and cities along both banks of the Yangtze River. When the Yangtze River waterway is regulated, it is necessary to minimize the negative effects of the project on the levees and flood control and at the same time the effects of the project on the ecological environment and the residents along both banks should be considered. Before the regulated schemes are made, all the factors should be considered so as to minimize the negative effects while the navigation conditions are improved. The mathematical model, as a means of research, will play more and more important roles in the forecast of the channel condition and the river change as well as the demonstration of the improvement schemes.

Table 4 Comparison of Scheme II and Scheme IV

Items to be Compared		Scheme II	Scheme IV	Conclusion
Navigation Condition	Channel Dimension	Meet the demand for the two-way navigation	Meet the demand for the two-way navigation. The bend radius of the entrance of Donggang branch is bigger than that in Scheme II	Scheme IV is superior to Scheme II
	Flow Condition at the Entrance of Donggang Branch	1.The scope of backflow at the entrance of Donggang branch reduces in dry season. 2.The flow is a little bit smooth with an angle of 4° to 7° to the left than before the project.	1.The backflow at the entrance of Donggang branch disappears in dry season; 2.The flow moves more smoothly with an angle of 6° to 9° to the left than before.	Scheme IV is superior to Scheme II
Effect on the River Channel	North Channel	1.At the discharge of 35,300m ³ /s,the diversion ratio reduces by 0.15%; 2.At the discharge of 12,050m ³ /s,the diversion ratio reduces by 0.36%.	1. At the discharge of 35,300m ³ /s, the diversion ratio reduces by 0.19%; 2. At the discharge of 12,050m ³ /s, the diversion ratio reduces by 0.44%	Scheme IV is beneficial to the safety of Zongyang Levee in the North Channel
	Entrance of Donggang Branch	1.At the discharge of 35,300m ³ /s, the diversion ratio increases by 1.17%; 2.At the discharge of 12,050m ³ /s, the diversion rate increases by 0.58%.	1.At the discharge of 35,300m ³ /s, the diversion ratio increases by 1.4%; 2.At the discharge of 12,050m ³ /s, the diversion rate increases by 0.5%.	Nearly the same
	An'ao Channel	1.At the discharge of 35,300m ³ /s, the diversion ration decreases by 0.3%; 2.At the discharge of 12,050m ³ /s, the diversion ratio basically has no change.	1.At the discharge of 35,300m ³ /s, the diversion ration decreases by 0.36%; 2.At the discharge of 12,050m ³ /s, the diversion ratio basically has no change.	Scheme IV is a bit better
	Xigang Branch	Basically there is no change.	Basically there is no change.	Nearly the same
	Overbank Flow on Central Bar	1.At the discharge of 35,300m ³ /s, the diversion ratio decreases by 0.73%; 2.At the discharge of 12,050m ³ /s, the diversion ratio increases by 0.25%.	1.At the discharge of 35,300m ³ /s, the diversion ratio decreases by 0.86%; 2.At the discharge of 12,050m ³ /s, the diversion ratio decreases by 0.2%.	Scheme IV is a bit better than Scheme II.

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