1-D SEDIMENT NUMERICAL MODEL AND ITS APPLICATION

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ABSTRACT

A one dimensional sediment numerical model SUSBED-2 has been developed to predict the aggradation and degradation of riverbed based on energy equation of the steady flow, nonuniform sediment transport equations for suspended load and bedload, sediment continuity equation and riverbed material composition equation. An improvement method to solve sediment transport equation of suspended load make the model can get better results for fast expanding or narrowing river reaches and for the reaches with lateral inflow. The model has been verified by field measurements and the results are generally fitted well in with the measurements. The model has been used in the design work of more than 40 projects of hydropower and water resources.

1. INTRODUCTION

To achieve the most efficiency in reservoir design, it is very important to predict the sediment deposition and its affects on the engineering, and to adjust the storage level and reservoir operation in accordance with the results of prediction. Riverbed degradation downstream of dam due to large decrease of sediment supply is also considered to alleviate possible engineering problems. As faster、more economic and more efficient tools, one dimensional numerical models have been widely used in the design work of reservoirs with different scales to replace former empirical prediction methods. In present paper, a one dimensional numerical called SUSBED-2 is developed based on the energy equation of steady flow and sediment transport equations for both suspended load and bedload. Sediment particles are divided into several groups according to their sizes that have different setting velocity parameters in related equations of sediment transport. Bed layer model(Wu,1994) is introduced into SUSBED-2, which can be used to calculate sediment deposition and riverbed scour, to simulate exchange between particles in flow and bed materials.

Han Qiwei(1980) developed the equation of nonequilibrium sediment transport of suspended load with nonuniform sizes and got two analytical solutions by the simplification of the equation respectively. This equation and its two solutions are widely and well used to solve sediment transport problems, but for fast expanding or narrowing river reaches, the calculations of the two solutions present unreasonable results because of its great differences between the simplification

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and real condition. An improved method to solve the nonequilibrium sediment transport equation is developed in SUSBED-2.

2. GOVERNING EQUATIONS

The basic one dimensional equations for steady flow and for nonequilibrium sediment transport processes can be expressed as following(Yang,1994):

Continuity equation for water

$$\frac{\partial Q}{\partial x} = q_1 \tag{1}$$

Energy equation for water

$$\frac{\partial Z}{\partial x} + \frac{1}{2g}\frac{\partial}{\partial x}\left(\frac{Q^2}{A^2}\right) + \frac{n^2 Q^2}{A^2 R^{4/3}} = 0 \tag{2}$$

Continuity Equation for sediment

$$(1-p_s)\frac{\partial A_{sk}}{\partial t} + \frac{\partial (QS_k)}{\partial x} + \frac{\partial G_k}{\partial x} = q_s + q_b \tag{3}$$

Nonequilibrium sediment transport Equation for suspended load

$$\frac{\partial (QS_k)}{\partial x} = -\alpha \omega_k B(S_k - S_{*k}) + q_s \tag{4}$$

For bedload both equilibrium nonequilibrium sediment transport Equations are considered, the equilibrium equation is

$$G_k = G_{*k} \tag{5}$$

And the nonequilibrium equation is

$$\frac{\partial G_k}{\partial x} = -K_t(G_k - G_{*k}) + q_b \tag{6}$$

Riverbed material composition equation(Wu，1994)

$$\frac{\partial (H_m P_k)}{\partial t} + \frac{1}{(1-p_s)B}\left[\frac{\partial (QS_k)}{\partial x} + \frac{\partial G_k}{\partial x}\right] + \left[\varepsilon P_k + (1-\varepsilon)P_{ok}\right]\left(\frac{\partial Z_s}{\partial t} - \frac{\partial H_m}{\partial t}\right) = 0 \tag{7}$$

In above equations *Q*=water discharge; *Z, A, R* and B=water level, flow area, hydraulic radius and width, respectively; x=distance along channel; g=gravitational acceleration； *n*=Manning coefficient of roughness; q_l , q_s q_b =lateral water inflow, suspended load and bedload discharges in unit length, respectively, for single channel sub-reaches, q_l , q_s , q_b =0; S_k , S_{*k} =sediment concentration and sediment capacity by volume, respectively; G_k and G_{*k}=bedload discharge by volume for equilibrium and nonequilibrium state, respectively; p_s =porosity of sediment in bed layer; ω_k =setting velocity; α =the saturation recovery coefficient for suspended load; K_t =the saturation recovery coefficient for bedload; P_k=bed layer composition at any time; P_{ok} =original bed layer composition; Z_s= bed level; H_m=thickness of bed layer; during calculation, ε =0 when bed level lower than original river bed due to scour and ε =1 in other cases. Subscript *k* denotes the size group sequence for nonuniform sediment. For total suspended load and bedload, there exist following relations:

$$S = \sum_k S_k \tag{8}$$

$$S_* = \sum_k S_{*k} \tag{9}$$

$$G = \sum_k G_k \tag{10}$$

$$G_* = \sum_k G_{*k} \tag{11}$$

3. NUMERICAL SCHEME

Eqs.1 、2 and 3 are discretized as follows

$$Q_{i+1} = Q_i + Q_l \tag{12}$$

$$Z_j = Z_{j+1} + \frac{\Delta x n^2 \overline{Q}^2}{\overline{A}^2 \overline{R}^{4/3}} + \frac{1}{2g}\left[\left(\frac{Q^2}{A^2}\right)_{j+1} - \left(\frac{Q^2}{A^2}\right)_j\right] \tag{13}$$

$$(1-p_s)\left[\psi \Delta A_{skj+1} + (1-\psi)\Delta A_{skj}\right]\Delta x = \left(Q_{kj}S_{kj} - Q_{kj+1}S_{kj+1} + G_{kj} - G_{kj+1}\right)\Delta t \tag{14}$$

in which Δx =length increments and j=section sequence along distance, ΔA_{sj} and ΔA_{sj+1} are deposition or scour areas in section j and j+1 respectively, weighting factor $\Psi > 0.5$, $\overline{Q}$ 、 $\overline{A}$ and $\overline{R}$ are expressed as

$$\overline{Q} = (1-\Psi)Q_j + \Psi Q_{j+1} \tag{15}$$

$$\overline{A} = (1-\Psi)A_j + \Psi A_{j+1} \tag{16}$$

$$\overline{R} = (1-\Psi)R_j + \Psi R_{j+1} \tag{17}$$

To get ΔA_{sj} at inlet sections both for main channel and for branches, which should be determined before the solution of Eq.14, substitute $\frac{\partial(QS_k)}{\partial x}$ and $\frac{\partial G_k}{\partial x}$ into Eq.3 with Eqs.4 and 6, Eq.3 is changed to

$$(1-p_s)\frac{\partial A_{sk}}{\partial t} = \alpha\omega_k B(S_k - S_{*k}) + K_k(G_k - G_{*k}) \tag{18}$$

Eq.18 describes the relation of variable in single section, so it can be directly discretized as

$$\Delta A_{skj} = \frac{\Delta t}{\gamma'}\left[\alpha\omega_k B_j\left(S_{k_j} - S_{*kj}\right) + K_k\left(G_{kj} - G_{*kj}\right)\right] \tag{19}$$

For the calculation of deposition area of cross-sections, Eq.18 is used at inlet sections and Eq.14 for other sections

The solution(Yang, 1994) of Eq.6 is

$$G_{j+1} = G_{sj+1} + \left(G_j - G_{sj}\right)e^{-K_t\Delta x} + \left(q_b + \frac{G_{sj} - G_{sj+1}}{\Delta x}\right)\frac{1}{K_t}\left(1 - e^{-K_t\Delta x}\right) \tag{20}$$

Eq.7 is discretized as

$$\begin{cases} \psi\Delta\left(P_k H_m B_m\right)_{j+1} + (1-\psi)\Delta\left(P_k H_m B_m\right)_j + \dfrac{\Delta G_{Tk}\Delta t}{\Delta x} + \psi\left[\varepsilon P_{0k} + (1-\varepsilon)P_k\right]_{j+1}\left(\Delta A_{sj+1} - \Delta H_{mj+1}B_{mj+1}\right) \\ + (1-\psi)\left[\varepsilon P_{0k} + (1-\varepsilon)P_k\right]_j\left(\Delta A_{sj} - \Delta H_{mj}B_{mj}\right) = 0 \end{cases} \tag{21}$$

In which

$$\Delta G_{Tk} = G_{kj+1} - G_{kj} + \left(QS_k\right)_{j+1} - \left(QS_k\right)_j \tag{22}$$

$$\Delta\left(P_k A_m\right) = \left(P_k A_m\right)^{n+1} - \left(P_k A_m\right)^n \tag{23}$$

4. SUSPENDED LOAD CALCULATION

Eq.4 can bed written as:

$$\frac{\partial S_k}{\partial x} + \frac{1}{Q}\left(\frac{\partial Q}{\partial x} + \alpha B\omega_k\right)S = \frac{1}{Q}\left(q_s + \alpha B\omega_k S_{*k}\right) \tag{24}$$

It is a first order linear differential equation and its general solution is

$$S_k = e^{-\int P_k dx}\left(\int F e^{-\int P_k dx} dx + C_1\right) \tag{25}$$

In which $$P_k = \frac{1}{Q}\left(\frac{\partial Q}{\partial x} + \alpha B \omega_k\right) \tag{26}$$

And $$F_k = \frac{1}{Q}\left(q_{s_k} + \alpha B \omega_k S_{*k}\right) \tag{27}$$

Use the boundary condition to eliminate constant C_1, i.e.

$$S_{0k}\left[e^{\int P_k dx}\right]_{x=0} = \left[\int F_k e^{\int P_k dx}\right]_{x=0} + C_1, \qquad \text{When } x=0 \tag{28}$$

So, Eq.25 becomes

$$S_k = S_{0k}\exp\left(-\int_0^x P_k dx\right) + \exp\left(-\int P_k dx\right)\int_0^x \left[F_k \exp\left(\int P_k dx\right)\right] dx \tag{29}$$

In which S_{0k} is the sediment concentration at x=0. Generally it is impossible to integrate above equation because of variation of river width B and sediment capacity S_{*k} with distance x. Han Qiwei(1994) has developed the simplified solutions of Eq.29 for the simple sub-reaches without lateral inflow, in which $\frac{\partial Q}{\partial x} = 0$ and $q_s = 0$. By assuming unit flow discharge $q = \frac{Q}{B}$=constant and linear variation of S_{*k} with x, Eq.29 is integrated as

$$\mathrm{S}_k = \mathrm{S}_{*k} + (\mathrm{S}_{0k} - \mathrm{S}_{0*k})e^{-\frac{\alpha\omega_k \Delta x}{q}} + \left(S_{0*k} - S_{*k}\right)\frac{q}{\alpha\omega_k \Delta x}\left(1 - e^{-\frac{\alpha\omega_k \Delta x}{q}}\right) \tag{30}$$

In which, Δx =space interval, S_{0*k} and S_{*k} are sediment carrying capacity at x=0 and x respectively, by assuming q=constant and S_* =constant, Eq.29 is integrated as

$$\mathrm{S}_k = \overline{\mathrm{S}}_{*k} + (\mathrm{S}_{0k} - \overline{\mathrm{S}}_{*k})e^{-\frac{\alpha\omega_k \Delta x}{q}} \tag{31}$$

In which $\overline{\mathrm{S}}_{*k}$ is average sediment carrying capacity in sub-reach.

The early version of susbed-2 uses Eq.30 to calculate sediment transportation of suspended load. As it is pointed out in the introduction, improvement of Eqs.30 and 31 is necessary for fast expanding or narrowing river reach, in which there exists great difference between real situation and Han's assumptions about q and S_{*k}. A new method is developed to improve Han's solution as well as to apply it to the sub-reach with lateral inflow.

Although river width B is generally uncertain for natural river, calculation sections can be chosen carefully to keep its monotone variation in each sub-reach. In this case B can be generalized as linear variation in the reach because of its not very long distance, i.e.

$$B = B_j + \frac{x}{\Delta x}\left(B_{j+1} - B_j\right) \tag{32}$$

In which B_j and B_{j+1} are river widths at upper and lower sections of the sub-reach respectively. So Eqs.26 and 27 become

$$P_k = \frac{1}{Q}\left\{\frac{\partial Q}{\partial x} + \alpha\omega_k\left[B_0 + \frac{x}{\Delta x}\left(B_{j+11} - B_j\right)\right]\right\} \tag{33}$$

$$F_k = \frac{1}{Q}\left\{q_{s\,k} + \alpha\omega_k S_{*k}\left[B_0 + \frac{x}{\Delta x}\left(B_{j+1} - B_j\right)\right]\right\} \tag{34}$$

$\int P_k dx$ In Eq.29 can be integrated for two cases

(1) For reaches without lateral inflow, i.e. $\frac{\partial Q}{\partial x} = 0$, $q_l = 0$ and $q_s = 0$

$$\int P_k dx = \frac{\alpha\omega_k}{Q}\left(\frac{1}{2}\frac{B_{j+1} - B_j}{\Delta x}x^2 + B_j x\right) + C_2 \tag{35}$$

(2) For reaches with lateral inflow, i.e. $\frac{\partial Q}{\partial x} \neq 0$, $q_l \neq 0$ and $q_s \neq 0$

$$\int P_k dx = \left(\alpha\omega_k \frac{B_{j+1} - B_j}{Q_{j+1} - Q_j}x\right) + \left(M_k + 1\right)\ln Q + C_3 \tag{36}$$

In which

$$M_k = \frac{\alpha\omega_k \Delta x\left(B_j Q_{j+1} - B_{j+1}Q_j\right)}{\left(Q_{j+1} - Q_j\right)^2} \tag{37}$$

And

$$Q_{j+1} = Q_j + q_l \Delta x \tag{38}$$

In SUSBED-2 Simpson’s rule is adopted to make numerical integration of Eq.33, because it is difficult to get its analytical solution for above both cases. S_{*k} can be calculated by the interpolation values of related hydraulic factors and Han’s assumption of linear variation of S_{*k} is not longer needed.

5 SUPPLEMETAY RELATIONS AND PARAMETERS

Sediment carrying capacity is calculated in following formula (Yang,1994)

$$S_{*k} = \beta_{*k} S_{*k}\left(d_k\right) \tag{39}$$

$S_{*k}\left(d_k\right)$ is uniform sediment carrying capacity. According to Zhang Rijin(1997), $S_{*k}\left(d_k\right)$ is expressed as

$$S_{*k}\left(d_k\right) = K\left(\frac{V^3}{gR\omega_k}\right)^m \tag{40}$$

V is mean section velocity, K and m are parameters that are determined by measurement data, it is suggested that K=0.124 and m=1.05 based on the calibration with field measurement data from Gongzui, which will be introduced in following. Coefficient β_{*k} is expressed as

$$\beta_{*k} = \frac{\left(\frac{p_k}{\alpha\omega_K}\right)}{\sum_k \left(\frac{p_k}{\alpha\ \omega_k}\right)} \tag{41}$$

In which p_k is sediment composition of size group k of bed-material, α =1 for scour and α =0.25 for deposition.

Bedload discharge for each size group G_{*k} is calculated by

$$G_{*k} = p_k B g_b(dk) \tag{42}$$

In which $g_b(dk)$ is bed load rate by volume and is calculated by Meyer-Peter(1948) and Muller's formula. The value of K_t in Eq.6 is 0.002.

The thickness of bed layer H_m is estimated as 2m for reservoir and is calculated in other cases by

$$H_m = C_u H \tag{43}$$

In which H is water depth and C_u =0.2-0.3

6 VERIFICATION AND APPLICATION

The filed measurement data from Gongzui is chosen to verify SUSBED-2. Gongzui Reservoir on Daduhe River, a sub-branch of upper Changjiang River in Sichuan Province of China, was built up in 1971 and measurement in the reservoir was begun in 1967. From 1977, former Ministry of Water Resources and Electric Power chose it as key reservoir for measurements and scientific investigation. Measurement items include sections along 41.37m long distance, bed material sample and analysis, water levels at 11 positions and measurements of outlet of water and sediment from the reservoir. In upper reach of the reservoir there exists a formal hydrological station, which controls inlet of the reservoir.

The calculation period is nearly 10 years from May 1, 1977 to Dec.31, 1986 and the initial sections of the reservoir is measured in April 1977. The average runoff of suspended load is

$39.6\times10^6\,t$ and average concentration of suspended load is $0.824\text{kg}/\text{m}^3$. Average bedload runoff is estimated as $0.88\times10^6\,t$ based on flume experiments by former Chengdu Institute of Investigation and Design. The value of p_s is 0.47.

Part of results is given in comparison with measurements in Table 1, 2 and Figure 1, 2 respectively. Calculation results are generally well fitted in with the field data in consideration of accuracy of measurements.

Table 1 Comparison of Yearly Deposition Volume

Year	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
Measurement (10^6m^3)	12.76	27.89	43.13	59.37	72.92	81.07	97.16	102.30	108.77	109.26
Calculation (10^6m^3)	10.77	27.93	45.96	62.73	83.09	94.03	104.40	112.50	117.00	121.90
Relative Error (%)	-15.6%	0.1%	6.6%	5.7%	13.9%	16.0%	7.5%	10.0%	7.6%	11.6%

Table 2 Comparison of Yearly Average Outlet Sediment Size

Year		1978	1979	1980	1983	1984	1985	1986
Measurement	D_{50} (mm)	0.013	0.013	0.013	0.022	0.029	0.042	0.06
Calculation		0.008	0.013	0.013	0.025	0.031	0.048	0.049

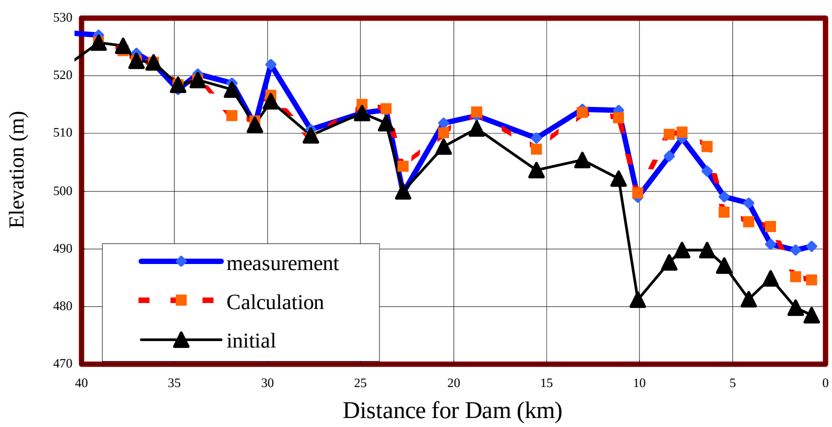

Figure 1 Comparison of Longitudinal Profiles in Nov. 1980

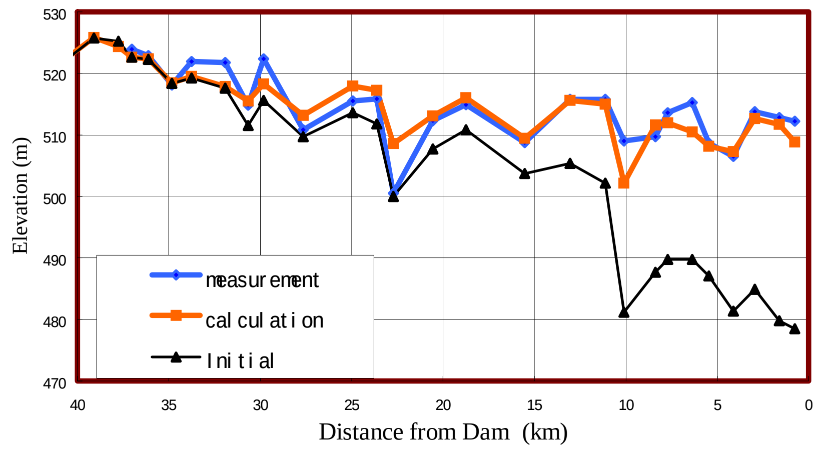

Figure 2 Comparison of Longitudinal Profiles in Nov. 1986

In 1996 the experts form former Hydropower and Water Resources Planning and Design General Institute of the Energy Ministry of China examined calculation results for several projects by SUSBED-2 and gave a positive appraisal. After that the model has been widely used to calculate river deformation in designing works for hydropower projects as a method to evaluate influences of sediment problems on the engineering after built-up of dams. As one of recommended sediment numerical models in Sediment Design Standard for Water Resources and Hydroelectric Engineering (DL/T 5080-1999 China), as many as 15 different design institutions up to now have used SUSBED-2 in the designing work of more than 40 projects of hydropower and water resources， including 8 in Jinshajiang River, the upper reach of Changjiang River and 2 in the upper of Yellow River。 SUSBED-2 is also compiled to Dynamical Energy Design Manual (Sediment Subdivision, which is about to be published) by Ministry of Water Resources.

7. EXAMPLE

One of applications of SUSBED-2 is to calculate sediment deposition of three huge hydropower projects, which are Baihetan, Xiluodu and Xiangjiaba from upper to lower respectively in lower reaches of Jishajiang River, and to analyze their influences on reduction of input sediment of Three Gorge reservoir, which is in lower of Jinshajiang and 433 km long from Xiangjiaba. The storage capacity of 4 reservoirs is listed in Table 3. The calculations are carried out for 200 years in the assumption that three Projects begin to operate at the same time.

Table 3 The Storage Capacity of 4 Projects

Projects	Baihetan	Xiluodu	Xiangjiaba	Three Gorge
Capacity of Reservoir ($10^8 m^3$)	173.34	115.7	49.77	393

The results in Figure 3 shows that deposition in Baihetan will not reach balanced state until 200 years later, and that for two other lower projects reservoir life can last much longer.

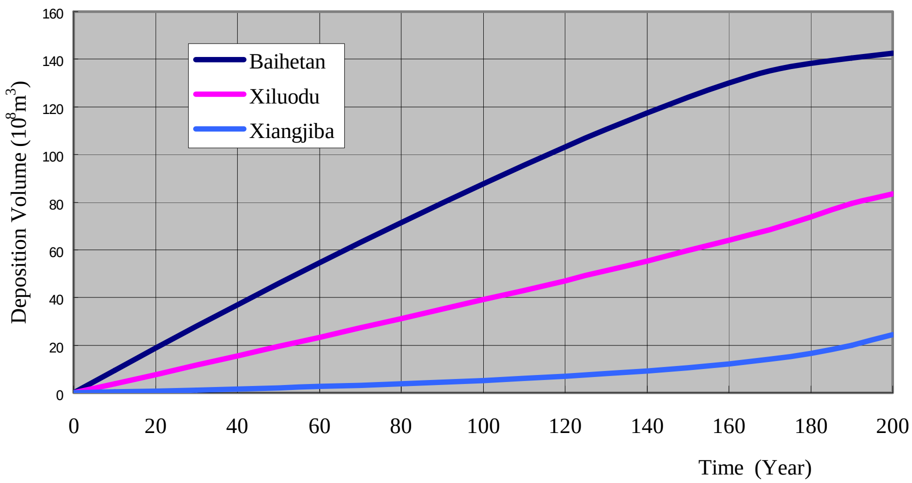

Figure 3 Deposition Volumes in Three Reservoirs

The outlet of Jinshajiang will delivery yearly 2.47×10^8 tons of sediment to the lower river before the construction of the three Projects. All of the sediment will enters into Three Gorge reservoir and 53.4% of input sediment of Three Gorge, which is about 4.6 2.47×10^8 tons yearly, is comes from Jinshajiang. When the Three projects begin to operate most of sediment and nearly all of the sediment with the size greater than 0.025mm are intercepted in the reservoirs. The decreases of sediment into the Three Gorge reservoir are given in Table 4 without considering sediment recovery between Xiangjiaba and inlet of the Three Gorge reservoir.

Table 4 Three Reservoirs Delivery Sediment and Reduction of Three Gorge Input Sediment

Years	Output Sediment in 10 years	Sediment Delivery Ratio	Yearly Average Reduction of Output sediment	Reduction Fraction of Tree Gorge Input Sediment in 10 Years	Accumulated Reduction of Three Gorge Input Sediment
（年）	（10^8t）	（%）	（10^8t）	（%）	（%）
0～10	4.36	17.65	2.03	44.22	44.22
11～20	4.46	18.07	2.02	43.91	44.11
21～30	4.54	18.40	2.02	43.91	44.01
31～40	4.63	18.73	2.01	43.70	43.92
41～50	4.72	19.11	2.00	43.48	43.82
51～60	4.82	19.51	1.99	43.26	43.72
61～70	4.92	19.90	1.98	43.04	43.62
71～80	5.01	20.28	1.97	42.83	43.52
81～90	5.12	20.72	1.96	42.61	43.41
91～100	5.25	21.27	1.94	42.17	43.30

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