Proceedings of the Ninth International Symposium on River Sedimentation
October 18 – 21, 2004, Yichang, China

RIVER SEDIMENTATION AND MORPHOLOGY MODELING – THE STATE OF THE ART AND FUTURE DEVELOPMENT

Sam S.Y. WANG and Weiming WU
National Center for Computational Hydroscience and Engineering
The University of Mississippi
MS 38677, USA

Abstract: In this paper, the state-of-the-art technologies for river sedimentation and morphology modeling are comprehensively reviewed. Emphasis is on computational modeling, which includes mathematical models and closures, numerical methods, verification and validation, model integration, and applications. Several important areas of future research are recommended.

Keywords: River Sedimentation, Morphological Processes, Computational Modeling

1 INTRODUCTION

River sedimentation and morphological processes are among the most complex and least understood phenomena in nature. Due to the fact that they intimately affect on our living conditions, scientists and engineers have been looking for better tools to improve our understanding and enhance the quality of our lives ever since the beginning of human civilization. In early times, the research methodologies were primarily based on field observation and physical modeling. This lecture is focused on the methodologies of mathematical modeling, more precisely numerical-empirical modeling. Sometimes, it is simply referred to as "computational Modeling".

The initial attempts of significance in application of mathematical models in conjunction with empirical functions obtained from laboratory experiments to the investigation of river sedimentation and morphological processes can be found in the 1950s. The branch of research was intensified and broadened in the 1970s. Since then, a number of 1-D models (e.g., Cunge *et al*., 1980; Thomas, 1982; Rahuel *et al*., 1989; Wu and Vieira, 2002) were applied to sedimentation studies in reservoirs and rivers. More recently, numerous 2-D and 3-D numerical-empirical models (e.g., Sheng, 1983; Wang and Adeff, 1986; Spasojevic and Holly, 1993; Jia and Wang, 1999; Wu *et al*., 2000) have been developed to simulate sediment transport processes and morphological changes in channels with mobile bed and bank, both in the laboratory and nature.

Like any other science and technology, the river sedimentation modeling has advanced from the simplified cases to the more and more complex ones with higher and higher levels of sophistication. In the early development stage, the sediment transport model only considered either bed load or suspended load with a single representative size. Realistically, sediments in natural rivers are often heterogeneous (nonuniform) in size, not to mention that they have variable shape and density. Furthermore, the transport mode changes from bed load to suspended load or vice versa with flow conditions. Consequently, the single-size, single-mode sediment transport model without considering the exchange between bed load and suspended load, nor the hiding/exposure and armoring mechanism has been found highly inadequate. Therefore, the research of sediment transport models was advanced in the direction of considering total load of nonuniform sediments.

Traditionally, the transport of sediment, especially bed load, was simulated based on the assumption of local or instantaneous equilibrium (saturation) (Thomas, 1982; Spasojevic and Holly, 1993). Due to the observed spatial and temporal lags between the sediment motion and water flow, it was found that the traditional equilibrium transport models need to be improved to take into account the non-equilibrium features of sediment transport, especially under unsteady flow conditions. Some examples of models with varying level of complication in simulating non-equilibrium (non-saturated) transport capability have been reported recently (Bell and Sutherland, 1983; Armanini and di Silvio, 1988; Rahuel *et al.*, 1989; Wu *et al.*, 2000; Wu and Vieira, 2002).

To meet the needs in engineering practices, the capability of river sedimentation modeling have been further advanced remarkably in recent years. Many established models are capable of simulating non-cohesive and cohesive sediment transport, local scouring, channel widening and meandering, etc. In addition, the integration of channel and watershed models has also been investigated.

With the constraint of presentation time, this lecture can only address briefly the state of the art and the feasible future development in river sedimentation modeling.

2 MATHEMATICAL MODELS

The phenomena of flow and sediment transport in rivers are characterized by turbulence, free-surface variation, bed change, phase interaction, etc. A model capable of including all of these effects accurately has yet to be developed. At the present, most sediment transport models have adopted the following assumptions:

(1) Sediment concentration is so low that the interaction between flow and sediment movement can be neglected. Therefore, the clear-water flow and the sediment advection-diffusion equations can be solved separately.

(2) Bed change is much slower than flow movement. Therefore, at each time step the flow can be calculated assuming a "fixed" bed.

(3) The hiding and exposure mechanism in bed material is considered through the introduction of correction factors in the nonuniform sediment transport capacity formulas.The interactions among different size classes of moving sediment are ignored. Thus the transport ofeach size class of sediment can be handled individually.

Based upon the above assumptions, 1-D, 2-D and 3-D governing equations of flow and sediment transport are developed.

2.1 1-D Model Equations

In 1-D model, the shallow water flow is governed by the well-known St. Venant equations. The sediment transport can be separated as bed load and suspended load according to sediment transport modes, as shown in Fig.1, or as bed-material load and wash load according to sediment sources. The governing equation for the non-equilibrium transport of non-uniform sediment is

$$\frac{\partial(AC_{tk})}{\partial t}+\frac{\partial Q_{tk}}{\partial x}+\frac{1}{L_s}\left(Q_{tk}-Q_{t*k}\right)=q_{lk} \quad (k=1, 2,\ldots, N) \tag{1}$$

where t is the time; x is the longitudinal coordinate; C_{tk} is the section-averaged sediment concentration; Q_{tk} is the actual sediment transport rate; Q_{t*k} is the sediment transport capacity or the so-called equilibrium transport rate; L_s is the non-equilibrium adaptation length of sediment transport; and q_{lk} is the side inflow or outflow sediment discharge from bank boundaries or tributary streams per unit channel length; each k represents a sediment size class; N is the total number of size classes.

Eq. (1) can be applied to bed load, suspended load, bed-material load or wash load, depending on how the sediment transport rate and the adaptation length are defined. For example, defining $L_s = Uh/(\alpha\omega_{sk})$, $Q_{tk} = QC_{tk}$ and $Q_{*tk} = QC_{*tk}$, one can rewrite Eq. (1) as the commonly used suspended-load transport equation with the exchange term $\alpha\omega_{sk}B(C_{tk} - C_{t*k})$. Here, U is the section-averaged velocity, h is the flow depth, ω_{sk} is the settling velocity of sediment particles, α is the non-equilibrium adaptation coefficient or the recovery coefficient, and C_{*tk} is the suspended-load transport capacity. When Eq. (1) is applied to the bed-material load, the transport rate Q_{tk} is the sum of bed-load and suspended-load transport rates. Eq. (1) can be also applied to wash load, where the adaptation length L_s is assumed to be infinitely large and then the exchange term on the left-hand side is zero.

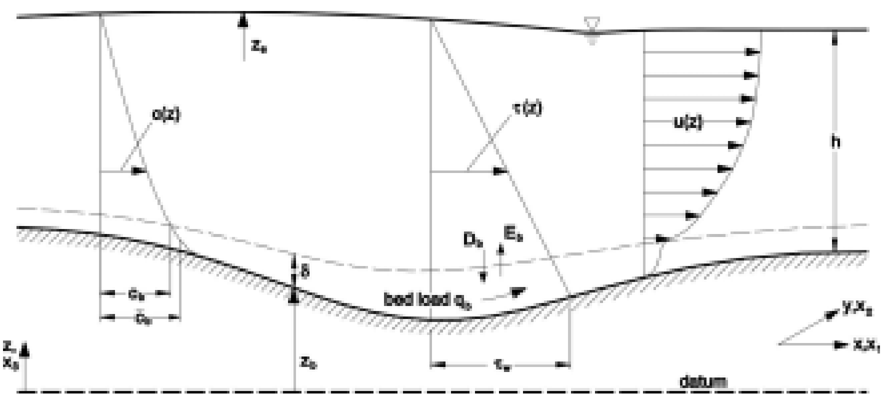


Fig. 1 Configuration of Flow and Sediment Transport

The bed deformation due to size class k is determined with

$$(1 - p')\left(\frac{\partial A_b}{\partial t}\right)_k = \frac{1}{L_s}\left(Q_{tk} - Q_{t*k}\right) \quad (k\text{=}1, 2,\ldots, N) \tag{2}$$

where p' is the bed material porosity; and $(\partial A_b / \partial t)_k$ is the bed deformation rate caused by size class k. In fact, combining Eqs. (1) and (2) leads to the sediment continuity equation, which could be also used to calculate the bed deformation.

It should be noted that as the sediment size increases, the adaptation length L_s reduces. As a result, the exchange term becomes dominant in Eq. (1). In such a case, Eq. (1) reduces to $Q_{tk} \approx Q_{t*k}$, which is the traditional assumption of local equilibrium. Therefore, the equilibrium sediment transport model is an extreme or special case of the non-equilibrium transport model. In the equilibrium transport model, the bed change is determined by the sediment continuity equation rather than Eq. (2).

The sediment transport capacity can be written in a general form as

$$Q_{t*k} = p_{bk} Q_{tk}^{*} \quad (k\text{=}1, 2,\ldots, N) \tag{3}$$

where p_{bk} is the availability factor of sediment, which is defined here as the bed material gradation in the mixing layer; Q_{tk}^{*} is the potential sediment transport capacity for size class k, which can be determined with the help of empirical relations to be given later.

To account for the variation of bed material gradation in time and space, the bed material is often divided into several layers at each computational node. The surface layer is the mixing layer that directly participates in the exchange with the sediment moving with the flow.

According to mass balance, the following equation for the variation of bed material gradation in the mixing layer (surface layer) can be derived (see Wu and Vieira, 2002)

$$\frac{\partial(\delta_m p_{bk})}{\partial t} = \left(\frac{\partial z_b}{\partial t}\right)_k + p^*_{bk}\left(\frac{\partial \delta_m}{\partial t} - \frac{\partial z_b}{\partial t}\right) \quad (k=1, 2, \ldots, N) \tag{4}$$

where p_{bk} is the bed material gradation in the mixing layer; δ_m is the mixing layer thickness, which is related to bed material size or sand dune height; $\partial z_b / \partial t$ is the total bed deformation rate, $\partial z_b/\partial t = \sum_{k=1}^{N}(\partial z_b/\partial t)_k$; p^*_{bk} is p_{bk} when $\partial\delta_m / \partial t - \partial z_b / \partial t \le 0$, and p^*_{bk} is the bed material gradation at the subsurface layer (below the mixing layer) when $\partial\delta_m / \partial t - \partial z_b / \partial t > 0$; The last term on the right-hand side represents the exchange between the mixing layer and subsurface layer.

The bed material sorting model in Eq. (4) is similar to Karim and Kennedy's (1982) mixing layer model, but different from Spasojevic and Holly's (1993) active layer model in which the active layer includes the bed load layer and the mixing layer.

2.2 2-D Model Equations

2-D models include two groups, the depth-averaged and width-averaged. The depth-averaged 2-D model is more often used in river engineering analysis. This model applies to simulating the shallow water flow and is governed by the depth-integrated continuity and Navier-Stokes equations:

$$\frac{\partial h}{\partial t} + \frac{\partial(hU)}{\partial x} + \frac{\partial(hV)}{\partial y} = 0 \tag{5}$$

$$\frac{\partial(hU)}{\partial t} + \frac{\partial(hUU)}{\partial x} + \frac{\partial(hVU)}{\partial y} = -gh\frac{\partial z_s}{\partial x} + \frac{1}{\rho}\frac{\partial(hT_{xx})}{\partial x} + \frac{1}{\rho}\frac{\partial(hT_{xy})}{\partial y} + \frac{\partial D_{xx}}{\partial x} + \frac{\partial D_{xy}}{\partial y} + \frac{1}{\rho}(\tau_{sx} - \tau_{bx}) + f_c hV \tag{6}$$

$$\frac{\partial(hV)}{\partial t} + \frac{\partial(hUV)}{\partial x} + \frac{\partial(hVV)}{\partial y} = -gh\frac{\partial z_s}{\partial y} + \frac{1}{\rho}\frac{\partial(hT_{yx})}{\partial x} + \frac{1}{\rho}\frac{\partial(hT_{yy})}{\partial y} + \frac{\partial D_{yx}}{\partial x} + \frac{\partial D_{yy}}{\partial y} + \frac{1}{\rho}(\tau_{sy} - \tau_{by}) - f_c hU \tag{7}$$

where x and y are the horizontal Cartesian coordinates; h is the flow depth; U and V are the depth-averaged flow velocities in x- and y-directions; z_s is the water surface elevation; g is the gravitational acceleration; T_{xx}, T_{xy}, T_{yx} and T_{yy} are the depth-averaged turbulent stresses; D_{xx}, D_{xy}, D_{yx} and D_{yy} are the dispersion terms due to the effect of secondary flow, which are important in the situations of curved channels; ρ is the density of fluid; τ_{bx} and τ_{by} are the bed shear stresses, determined by $\tau_{bx} = \rho c_f U\sqrt{U^2+V^2}$; $\tau_{by} = \rho c_f V\sqrt{U^2+V^2}$, with $c_f = gn^2/h^{1/3}$ and n is the Manning's roughness coefficient; τ_{sx} and τ_{sy} represent the shear forces acting on the water surface, usually caused by wind; f_c is the Coriolis coefficient.

The depth-averaged transport equation for suspended sediment is

$$\frac{\partial(hC_k)}{\partial t} + \frac{\partial(UhC_k)}{\partial x} + \frac{\partial(VhC_k)}{\partial y} = \frac{\partial}{\partial x}\left(\varepsilon_s h\frac{\partial C_k}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varepsilon_s h\frac{\partial C_k}{\partial y}\right) + \frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y} + \alpha\omega_{sk}(C_{*k} - C_k) \quad (k=1, 2, \ldots, N) \tag{8}$$

where C_k is the depth-averaged suspended-load concentration; C_{*k} is the suspended-load transport capacity; ε_s is the turbulence diffusivity coefficient of sediment, determined with $\varepsilon_s = \nu_t / \sigma_c$, in which σ_c is the turbulent Prandtl-Schmidt number, usually in 0.5-1.0 or determined by using van Rijn's (1989) method; α is the non-equilibrium adaptation coefficient.

In Eq. (8), S_x and S_y are the dispersion terms to account for the effect of the nonuniform distributions of flow velocity and sediment concentration. In the nearly straight (or slightly curved) channels with simple geometry, the dispersion terms are usually neglected, or combined with either the convection terms by introducing a correction factor or the diffusion terms by adjusting the diffusivity coefficient (also called the mixing coefficient). In curved channels, the dispersion terms become more important, which will be discussed in Section 3.8.

The bed-load transport is determined by

$$\frac{\partial(\delta_b \bar{c}_{bk})}{\partial t} + \frac{\partial(\alpha_{bx} q_{bk})}{\partial x} + \frac{\partial(\alpha_{by} q_{bk})}{\partial y} + \frac{1}{L_s}(q_{bk} - q_{b*k}) = 0 \quad (k=1, 2,..., N) \tag{9}$$

where $\bar{c}_{bk}$ is the average concentration of bed load at the bed-load zone; α_{bx} and α_{by} are direction cosine components of bed-load movement, which is assumed to be along the direction of bed shear; q_{bk} is the bed-load transport rate of k-th size class; q_{b*k} is the corresponding bed-load transport capacity.

The bed deformation can be calculated using the overall sediment balance equation or the following equation

$$(1 - p'_m)\left(\frac{\partial z_b}{\partial t}\right)_k = \alpha\omega_{sk}(C_k - C_{*k}) + (q_{bk} - q_{b*k})/L_s \quad (k=1, 2,..., N) \tag{10}$$

To close the set of Eqs. (8)-(10), formulas for the suspended-load and bed-load transport capacities C_{*k} and q_{b*k} are needed. These formulas can be written in a general form

$$C_{*k} = p_{bk} C_k^*, \quad q_{b*k} = p_{bk} q_{bk}^* \quad (k=1, 2,..., N) \tag{11}$$

where p_{bk} is the bed material gradation at the mixing layer; C_k^* is the potential transport capacity of size class k of suspended load; and q_{bk}^* is the potential transport capacity of size class k of bed load.

The variation of bed material gradation in the mixing layer in a depth-averaged 2-D model is determined by Eq. (4) at each computational node.

2.3 3-D Model Equations

Generally, the 3-D flow field is determined by the following Reynolds-averaged continuity and Navier-Stokes equations:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{12}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial(u_i u_j)}{\partial x_j} = F_i - \frac{1}{\rho}\frac{\partial p_i}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j} \tag{13}$$

where u_i (i=1, 2, 3) are the velocity components; F_i includes the external forces, including the gravity force, per unit volume; p is the pressure; τ_{ij} are the turbulent stresses, which should be determined by using a turbulence model.

For the shallow water flow, the pressure can be assumed to be hydrostatic and all the vertical components of fluid acceleration can be ignored, thus yielding the quasi 3-D governing equations as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{14}$$

$$\frac{\partial u}{\partial t}+\frac{\partial (uu)}{\partial x}+\frac{\partial (vu)}{\partial y}+\frac{\partial (wu)}{\partial z}=-g\frac{\partial z_s}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{xx}}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{xz}}{\partial z}+fv \tag{15}$$

$$\frac{\partial v}{\partial t}+\frac{\partial (uv)}{\partial x}+\frac{\partial (vv)}{\partial y}+\frac{\partial (wv)}{\partial z}=-g\frac{\partial z_s}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{yx}}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{yy}}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{yz}}{\partial z}-fu \tag{16}$$

where f is the Coriolis coefficient.

The hydrostatic pressure assumption brings significant simplification to the full three-dimensional problem of Eqs. (12) and (13). However, this assumption is valid only for the gradually varying open-channel flows. A full 3-D model without hydrostatic pressure assumption should be used in the regions of the rapidly varying flows, such as flows around bridge piers, dikes, bendway weirs, etc. Since most environmental flows in rivers, estuaries and coastal zones can be assumed as shallow water flows, the hydrostatic pressure assumption has been often adopted. The 3-D models developed by Sheng (1983), Wang and Adeff (1986), and Casulli and Cheng (1992) are based on the hydrostatic pressure assumption, while those developed by Wu *et al*. (2000), and Jia *et al*. (2001) are not.

As shown in Fig. 1, the sediment transport is divided into suspended load and bed load and hence the flow domain is divided into a bed-load layer with a thickness δ and the suspended-load layer above it with a thickness $h-\delta$. The exchange of sediment between the two layers is through deposition (downward sediment flux) at a rate of D_{bk} and entrainment from the bed-load layer (upward flux) at a rate of E_{bk}. The distribution of the sediment concentration in the suspended-load layer is governed by the following convection-diffusion equation:

$$\frac{\partial c_k}{\partial t}+\frac{\partial\left[\left(u_j-\omega_{sk}\delta_{j3}\right)c_k\right]}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_c}\frac{\partial c_k}{\partial x_j}\right) \tag{17}$$

where c_k is the local concentration of the k-th size class of suspended load; δ_{j3} is the Kronecker delta with j=3 indicating the vertical direction.

At the free surface, the vertical sediment flux is zero and hence the condition applied is

$$\frac{\nu_t}{\sigma_c}\frac{\partial c_k}{\partial z}+\omega_{sk}c_k=0 \tag{18}$$

At the lower boundary of the suspended sediment layer, the deposition rate is $D_{bk}=\omega_{sk}c_{bk}$ while the entrainment rate E_{bk} is

$$E_{bk}=-\frac{\nu_t}{\sigma_c}\frac{\partial c_k}{\partial z}=\omega_{sk}c_{b*k} \tag{19}$$

where c_{b*k} is the equilibrium concentration at the reference level $z=z_b+\delta$, which needs to be determined using an empirical relation.

In the 3-D model, the bed-load transport is simulated by using Eq. (9), with α_{bx} and α_{by} being the direction cosines of the bed shear stress these cosines known from the flow calculation.

The bed change can be determined by either the exchange equation

$$\left(1-p'_m\right)\left(\frac{\partial z_b}{\partial t}\right)_k=D_{bk}-E_{bk}+\frac{1}{L_s}\left(q_{bk}-q_{b*k}\right) \tag{20}$$

or the overall sediment mass-balance equation integrated over the water depth h (i.e. from $z = z_b$ to z_s):

$$(1 - p'_m)\left(\frac{\partial z_b}{\partial t}\right)_k + \frac{\partial (hC_{tk})}{\partial t} + \frac{\partial q_{tkx}}{\partial x} + \frac{\partial q_{tky}}{\partial y} = 0 \tag{21}$$

where C_{tk} is the depth-averaged sediment concentration; and q_{tkx} and q_{tky} are the components of the total-load sediment-transport in x- and y-directions:

$$q_{tkx} = \alpha_{bx} q_{bk} + \int_\delta^h \left(u c_k - \frac{\nu_t}{\sigma_c} \frac{\partial c_k}{\partial x} \right) dz \; ; \quad q_{tky} = \alpha_{by} q_{bk} + \int_\delta^h \left(v c_k - \frac{\nu_t}{\sigma_c} \frac{\partial c_k}{\partial y} \right) dz \tag{22}$$

The bed material sorting in the 3-D model is computed in the same way as in the depth-averaged 2-D model.

3 MODEL CLOSURE AND AUXILIARY RELATIONS

3.1 Turbulence Model Closures

The turbulent shear stresses in 2-D and 3-D models need to be determined by turbulence models. Most of the common turbulence closures for river flow are based on the Bossinesq's eddy viscosity concept

$$\tau_{ij} = \rho \nu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij} \tag{23}$$

where k is the turbulent kinetic energy, which is omitted in the zero-equation turbulence models; ν_t is the eddy viscosity. The eddy viscosity is usually determined by the parabolic eddy viscosity model, the mixing length model and the linear k-ε turbulence model. The linear k-ε turbulence model includes the standard k-ε turbulence model, and the RNG k-ε turbulence model.

The Boussinesq's assumption, which adopts an isotropic eddy viscosity concept for all Reynolds stress, fails for the flows with sudden changes in mean-strain rate or with "extra" rates of strain, e.g. curvilinear flows. In such cases, the Reynolds stresses adjust to such changes at a rate unrelated to the mean flow processes and time scale. To capture this kind of turbulence-generated flow features, the non-linear k-ε turbulence model, the Reynolds-stress/flux model and the algebraic stress/flux model should be used (Rodi, 1993).

3.2 Channel Roughness

In natural rivers, the banks and bed usually have different roughness. The bank roughness elements include bank materials, channel training works, hydraulic structures, vegetation, etc., while the bed roughness elements include the rigid materials as well as the movable bed forms, such as sand ripples, sand dunes, alternate bars, islands, etc. For the banks and rigid bed, a constant roughness can be used. For the movable bed in an alluvial river, the bed roughness changes with flow conditions, so the evaluation of bed roughness is more difficult. Einstein and Barbarossa (1952), Engelund and Hansen (1967), van Rijn (1989), and Wu and Wang (1999) proposed empirical or semi-empirical methods to calculate the roughness in a movable bed. However, because these empirical relations usually rely on the data set used and may give different predictions at different sites or times, it should be cautioned when applying them in a site-specific study. The most reliable approach to handle the channel roughness is still the calibration using the available data measured at the study site.

More recently, investigators have tried to introduce a different modeling concept to account for the effect of vegetation roughness (Shimizu and Tsujimoto, 1994; Lopez and Garcia, 2001; Wu and Wang, 2004a). The drag force exerted on the vegetation element is considered in the momentum equations, and the generation of turbulence due to vegetation is modeled in the k and ε equations. After the effect of vegetation on flow is considered, its effect on sediment transport and channel morphology change can also be simulated.

3.3 Non-equilibrium Adaptation Length

The non-equilibrium adaptation length L_s, which characterizes the distance for sediment to adjust from a non-equilibrium state to an equilibrium state, is a very important parameter in the non-equilibrium transport model. Traditionally, the non-equilibrium adaptation length of suspended load is given with $L_s = Uh/(\alpha\omega_{sk})$. The coefficient α can be calculated with Armanini and di Silvio's (1988) method or other semi-empirical methods. Values of α calculated from these methods are usually larger than 1. However, in practice, α has been given different values by many researchers, most of them being less than 1. It has been suggested that α=1 for the case of strong erosion, α=0.25 for strong deposition, and α=0.5 for weak erosion and deposition, as a result obtained from validation tests in many reservoirs and rivers.

For bed load, Bell and Sutherland (1981) found that the L_s is a function of time t in an experimental case of bed degradation downstream of a dam due to clear water. In numerical modeling, Wu *et al*. (2000) have adopted L_s as the average saltation step length of sand on the bed in the laboratory cases, while Rahuel *et al*. (1989) used much larger values for L_s, which may be from one to two times of the numerical grid length, in the case of natural rivers. One reason for the disparity is that the non-equilibrium adaptation length L_s, especially for bed load, is closely related to the dimensions of the sediment movements under study, bed forms and channel geometry, which are significantly different between laboratory and field situations. In laboratory experiments, sediment transport processes are of small scales, such as sand saltation, ripples and dunes, while in streams sediment transport processes occur usually at larger scales in longer periods. More recently, Wu and Vieira (2002) suggested that the non-equilibrium adaptation length of bed load may be the same as the dimension of the dominant bed form, such as the length of sand dunes in laboratory flume cases, and the length of alternate bars in field cases. This suggestion has given very promising results in a series of applications.

3.4 Non-cohesive Sediment Transport Capacity Formulas

The sediment transport capacity formulas proposed by Meyer-Peter and Mueller (1948), Dou (1963), Yalin (1972), and van Rijn (1989) are often used to calculate the discharge of uniform bed load. These formulas are also capable of predicting the total discharge of nonuniform bed load. For fractional discharge of nonuniform bed load, the pioneering research was attributed to Einstein (1950), followed by Parker *et al*. (1982), Wu, Wang and Jia (2000), and others.

One of the most widely recognized methods for the suspended-load discharge is the Einstein's (1950) method, which integrates the product of sediment concentration c and flow velocity u over the suspended-load zone. The formulas of total discharge of suspended load proposed by van Rijn (1989) and Zhang (1959, see Zhang and Xie, 1993) are much simpler than that of Einstein's and have comparable reliability. Wu, Wang and Jia (2000) established a formula to directly calculate the fractional discharge of suspended load.

The total discharge of bed-material load can be determined by the methods proposed by Einstein (1950), Engelund and Hansen (1967), Ackers and White (1973), Yang (1973), and van Rijn (1989). For fractional discharge of nonuniform bed-material load, the modified Ackers and White's (1973) formula (Proffit and Sutherland, 1983), SEDTRA module (Garbrecht *et al*., 1995), and Wu, Wang and Jia (2000).

The equilibrium near-bed concentration of uniform suspended-load can be determined by using the methods proposed by van Rijn (1989) and Zyserman and Fredsφe's (1994). For the fractional near-bed concentration of nonuniform suspended load, the empirical formulas proposed by Einstein's (1950) and Garcia and Parker (1991) can be used. However, two issues should be noted. One is that the measurement of suspended-load concentration near the channel bed is very difficult. Usually, the near-bed concentrations have to be extrapolated from the measured sediment concentrations in the upper zone with the aid of an assumed concentration profile along the water depth. Therefore, the accuracy and reliability of this analysis are highly dependent on the selection of sediment concentration distribution near the bed. The other important issue is that different reference levels have been used to define the near-bed concentration in different formulas.

Because the sediment transport formulas having been developed are mostly empirical or semi-empirical and have large discrepancy among them, their calibration and validation are very important. The comparisons of the total-discharge formulas have been performed by Brownlie (1981), Yang and Wan (1991), and others. Most of these tests show that Ackers and White's (1973), Engelund and Hansen's (1967), Yang's (1973) and van Rijn's (1989) formulas are relatively more reliable for the total discharge of bed-material load. Ribberink *et al*. (2002) and Wu and Wang (2003) tested the formulas for the fractional discharge of nonuniform bed-material load, and found that Wu, Wang and Jia's (2000) formula and the SEDTRA module perform better. However, calibration and validation using site-specific data is highly recommended before applying a sediment transport formula to a study of real-life problems.

3.5 Cohesive Sediment Transport Formulas

Fine-grained sediments, such as clay and fine silt, which widely exist in rivers, reservoirs, lakes, estuaries and coastal waters, are generally cohesive. Because the magnitude of electrostatical forces acting among these particles is comparable to or larger than that of the gravity forces, the fine particles may stick together forming aggregates (or called flocs) when they collide due to Brownian motion of the particles (<0.004mm), the turbulent mixing and/or the settling. On the other hand, during the transport process, larger flocs may be disaggregated into finer flocs and single particles due to high shear or large eddy ejection and sweeping. Therefore, the flocs behave much differently from the non-cohesive single particles. Many investigators developed numerical models for cohesive sediment transport with special consideration of flocculation, settling, transport, erosion, deposition and consolidation processes.

The transport of cohesive sediment is usually treated as suspended load and governed by Eq. (17). The size of flocs and in turn the settling velocity is related to the particle size, sediment concentration, salinity, and turbulence intensity. Thorn (1981) and others established several empirical formulas for the settling velocity of the flocs. Generally, the settling velocity of the flocs can be determined by (Wu and Wang, 2004b)

$$\frac{\omega_f}{\omega_{d50}} = K_d K_s K_{sa} K_t \tag{24}$$

where ω_f is the representative settling velocity of the flocs; ω_{d50} is the settling velocity of single particles, corresponding to the median size d_{50} of the sediment mixture; K_d, K_s, K_{sa} and K_t are the correction factors accounting for the influences of sediment size, sediment concentration, salinity, and turbulence intensity, respectively.

Cohesive sediment is either eroded from the bed in particles (surface erosion) or in layers (mass erosion). The surface erosion occurs when the applied shear stress exceeds a certain critical shear stress, and mass erosion happens when the applied shear stress exceeds the bulk strength of the sediment. Partheniades (1965) found that the surface erosion rate is a linear function of the dimensionless excess shear stress, while Raudkivi and Hutchison (1974) observed that it is exponentially proportional to the dimensionless excess shear stress. Mehta (1986) clarified that the exponential relation is valid for partly consolidated beds and the linear relationship is valid for fully consolidated beds. Partheniades' (1965) erosion relation reads

$$E_b = M\left(\frac{\tau_b - \tau_{ce}}{\tau_{ce}}\right) \tag{25}$$

where M is a coefficient related to the material properties, such as mineral composition, organic material, salinity, etc.; and τ_{ce} is the critical shear stress for erosion, which depends on the mud dry density, organic material, temperature, pH value, the Sodium Absorption Ratio (SAR), etc.

Krone (1962) and Mehta and Partheniades (1975) investigated the deposition process of fine sediment, and proposed formulas to determine the deposition rate. Mehta and Partheniades' (1975) formula reads

$$D_b = \begin{cases} 0 & (\tau_b > \tau_{bd,\max}) \\ \alpha_m \omega_{s,m} C & (\tau_{bd,\min} < \tau_b \le \tau_{bd,\max}) \\ \omega_{s,m} C & (\tau_b < \tau_{bd,\min}) \end{cases} \tag{26}$$

where τ_b is the bed shear stress; $\tau_{bd,\min}$ is the critical bed shear stress below which all sediment are deposited on the bed; $\tau_{bd,\max}$ is the critical bed shear stress above which all sediment remain in suspension yielding a zero deposition rate; α_m is a coefficient between 0 and 1 related to the bed shear stress, and is approximated as $\alpha_m = 1 - (\tau_b - \tau_{bd,\min})/(\tau_{bd,\max} - \tau_{bd,\min})$.

Once the fine sediment particles deposit on the bed, consolidation will occur. The detailed consolidation process can be described by Gibson *et al.*'s (1967) theory. The model based on this theory calculates the evolution of the void ratio of a soil layer using a 1-D unsteady differential equation in the vertical direction. This approach is usually time-consuming. Therefore, a simpler approach is often used, which uses empirical functions, such as Lane and Koelzer's (1953), to determine the evolution of dry density. The decrease in bed elevation due to the consolidation is determined by

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\frac{H}{\bar{\rho}_d}\frac{\mathrm{d}\bar{\rho}_d}{\mathrm{d}t} \tag{27}$$

where H is the bed thickness; $\bar{\rho}_d$ is the mean dry density of the bed.

3.6 Local Scour near In-stream Structures

The local scour near in-stream structures such as bridge piers, abutments and spur-dykes is complex and difficult to model. The three-dimensional flow features, such as the downward flow, the localized pressure gradient, and horseshoe and wake vortices, are all important in the development of local scour. At the present, the highly complex, three-dimensional near-field flows can be calculated reasonably well with advanced turbulence models. However, how to quantify the local sediment transport mechanism near the structures is still a problem requiring further investigation.

To facilitate the design of in-stream structures, many empirical formulas, such as the HEC-18 (FHWA, 1995) equation for bridge pier scour, Froehlich's (FHWA, 1995) equation for bridge abutment scour, etc., have been developed and used with 1-D, 2-D and even 3-D flow models to predict the maximum scour depth at the structures.

To simulate the details of the erosion process around in-stream structures, especially under unsteady flow conditions, the sediment transport and bed morphology models have been developed. The governing equations are the same as those for the general sediment transport. The difference is in the sediment transport capacity formula. In the case of local scour simulation, one should select the formula that can account for the effect of downward flow, localized pressure gradient and fluctuations, vorticity, turbulence intensity, etc. Dou (1997), Jia *et al*. (2001) and Wu and Wang (2004c) have attempted to extend the capability of general sediment transport formulas for the local scour simulation near bridge piers, abutments and spur-dykes as well as the headcut. However, these empirical formulas need to be validated by more laboratory experiments and field measurements.

3.7 Sediment Transport on Steep Slopes

For the channels with steep slopes, the effect of the gravity on sediment transport can not be ignored. To consider the effect of the gravity on the sediment transport capacity expressed as $q_{b*} = f(\tau_b/\tau_c)$, one may use one of the two approaches. Here, τ_b is the bed shear stress, and τ_c is the critical shear stress for incipient motion of bed material. One approach is to correct the critical shear stress τ_c using the method of Brooks (1963) or van Rijn (1989). A disadvantage of this approach is that when the bed angle is equal to the repose angle, one must determine an accurate value for the critical shear stress so that the realistic q_{b*} is not infinite. Another approach is to add the streamwise component of the gravity force to the bed shear stress τ_b without modifying τ_c (Wu, 2004)

$$\tau_{be} = \tau_b + \lambda_0 \tau_c \sin\varphi / \sin\phi \tag{28}$$

where τ_{be} is the effective tractive force; φ is the bed angle with the horizontal, with positive values denoting downslope bed; ϕ is the repose angle; and λ_0 is a coefficient related to flow and sediment conditions as well as the bed slope (Wu, 2004). Eq. (28) can be used to modify many sediment transport formulas, such as van Rijn's (1989) formula and Wu *et al*.'s (2000) formula,

The effect of the gravity on the direction of bed-load transport has been investigated by Engelund (1974), Sekine and Parker (1992), and Wu (2004). To consider this, the parameters α_{bx} and α_{by} in Eq. (9) are replaced by $\alpha_{bx,e}$ and $\alpha_{by,e}$ which are determined as

$$\frac{\alpha_{bx,e}}{\alpha_{by,e}} = \frac{\alpha_{bx}\tau_b + \lambda_0\, \tau_c \sin\varphi_x / \sin\phi}{\alpha_{by}\tau_b + \lambda_0\, \tau_c \sin\varphi_y / \sin\phi} \tag{29}$$

where φ_x and φ_y are the bed angles along x- and y-directions.

3.8 Effect of Secondary Flow on Sediment Transport in Curved Bends

Helical (secondary) motions in curved channels play an important role in the evolution of channel morphology, inducing deposition along the inner bank and erosion along the outer bank. This phenomenon can be simulated by a three-dimensional model, which has been carried out by Wu *et al*. (2000) and others. However, for obtaining results quickly, a number of investigators, e.g., Flokstra (1977), Jin and Steffler (1993), and Wu and Wang (2004d) have modified the depth-averaged 2-D models to include the effect of the secondary helical motions. The dispersion terms in Eqs. (6)-(8) can be used to serve this purpose. Jin and Steffler (1993) calculated these terms by solving two extra equations that are obtained by integrating the 3-D horizontal moment-of-momentum equations over the depth. Flokstra

(1977) and Wu and Wang (2004d) determined these terms by using the following simple algebraic equation for the vertical distribution of the helical flow

$$u_n = U_n + b_s I\left(\frac{2z}{h} - 1\right) \tag{30}$$

and the Rouse distribution or Lane-Kalinske distribution for the suspended-load concentration along the depth. In Eq. (30), u_n is the local velocity in the cross-stream direction; b_s is the coefficient with a value of about 6.0; U_n is the depth-averaged velocity in the cross-stream direction; I is the intensity of helical flow. Theoretically, $I = U_s h/r$ in the channel centerline (Rozovskii, 1957). Here, r is the local radius of curvature. For the entire channel bend, de Vriend (1981) proposed a differential transport equation to determine the I. Wu and Wang (2004d) simplified this differential equation into an algebraic formula for the helical flow intensity in the fully developed region.

For the effect of the helical flow on the bed-load transport direction, the methods proposed by Engelund (1974) and Odgaard (1981) can be used.

3.9 Bank Erosion and Mass Failure

Bank erosion is the main cause of channel widening and meandering. To realistically model the morphological evolution of channels with movable banks, both bed and bank changes should be simulated (Duan *et al*., 2001; Wu and Vieira, 2002). Usually, for non-cohesive banks, the bank retreat occurs gradually when the slope angle is larger than the repose angle. For cohesive banks, bank material collapse in the form of mass failure, a discontinuous phenomenon. The bank instability and mass failures may be caused by bed degradation and lateral fluvial erosion at bank toes as well as the pore pressure change in the bank soil. Arulanandan *et al*. (1980) established an empirical relationship for calculating the erosion of bank toes. Osman and Thorne (1988) proposed an algorithm to analyze the stability of banks, with the safety factor being defined as the ratio of the resistance and driving forces for the failure. Simon *et al*. (2000) proposed a more sophisticated bank stability and toe erosion model, which considers wedge-shaped bank failures with several distinct bank material layers and user defined bank geometry. This model is able to incorporate root reinforcement and surcharge effects of six vegetation species, including willows, grasses and large trees, and simulate saturated and unsaturated soil strength considering the effect of pore-water pressure.

4 NUMERICAL METHODS

4.1 Discreization of Governing Equations

The numerical methods used in the discretization of flow and sediment transport equations include the finite element model, finite difference model, finite volume model, finite analytical model, efficient element method, etc. The basic information about these methods is available readily. In the case of the efficient element method, one can refer to Wang and Hu (1992).

The flow and sediment transport equations of the convection-diffusion type. The diffusion terms are usually discretized by using the central difference scheme or the similar ones. However, the convection terms have to be discretized by using the upwind schemes. The simplest upwind scheme is the first-order upwind scheme, which is very stable but has strong numerical diffusion. The often adopted numerical schemes with upwind features and acceptable numerical diffusion include the hybrid upwind/central scheme, exponential difference scheme (Spalding, 1972), the finite analytical scheme, and the upwind interpolation scheme (Wang and Hu, 1992). This group schemes usually have the second-order accuracy or less. Other upwind schemes, such as QUICK scheme (Leonard, 1979), have higher than second-order accuracy but may encounter numerical oscillation. Many limiters have been developed to control the potential numerical oscillation existed in these high-order upwind schemes.

For unsteady problems, the time derivative terms can be discretized by using either an explicit scheme or an implicit scheme. The explicit scheme results in simple algebraic equations and computer program. However, the time step in the explicit scheme is usually limited by the CFL condition, which means the Courant number should be less than 1. To allow a larger time step, the implicit scheme should be adopted. The overall efficiency of the implicit solution procedure depends on what kind of iteration solver is used. This will be discussed in the next section.

4.2 Iterative Solution of Algebraic Equations

Because the hydrodynamic problems are nonlinear, iterative methods are often used, especially for the solution of multidimensional problems. The simplest iteration methods are Jacobi method and Gauss-Seidel method. However, the convergence speed of these two methods is usually slow. To speed up the convergence, the Thomas algorithm and the Alternating Direction Implicit (ADI) iterative method are widely used.

Another iteration method of even faster convergence is the Strongly Implicit Procedure (SIP) proposed by Stone (1968). The idea is to approximately factorize the algebraic equations to two subsystems with a lower triangular matrix and an upper triangular matrix, and then solve the two subsystems separately by a direct method.

Other iteration methods include Newton's method, Conjugate Gradient method, Biconjugate Gradients method, multigrid method, etc. In addition, the relaxation method is often adopted to enhance the efficiency of iteration. Over- and under-relaxation methods can accelerate or slow down the convergence speed.

4.3 NUMERICAL SOLUTIONS OF FREE-SURFACE FLOW

4.3.1 One-Dimensional Flows

For 1-D steady or quasi-steady flow model, the governing equation is the energy equation, which can be solved by the standard step method. For 1-D unsteady model, the most widely used numerical scheme is Preismann's four-point implicit scheme. In a single channel or in a dendritic channel network, the resulting algebraic equations can be used by applying Thomas algorithm. In the case of looped channel networks, the method proposed by Cunge *et al*. (1980) is suggested.

The Preissman's scheme may encounter instability in the case of transcritical flow. Therefore, it is usually limited in the range of subcritical flow. For the flow mixed with subcritical flow and supercritical flow, the Riemann solver and TVD schemes are often used. Most recently, Ying *et al*. (2003) proposed an upwind scheme to determine the fluxes in the 1-D St. Venant equations, which can be used to solve the mixed flow, dam break flow, etc.

4.3.2 Two-Dimensional Flows

In the depth-averaged 2-D shallow water Eqs. (5)-(7), the water depth appears in the depth-integrated continuity Eq. (5), providing a strong linkage between velocity and pressure (water level). Usually, upwind schemes are needed to discretize the convection terms in the momentum equations and even the spatial derivative terms in the continuity equations (i.e., upwinding flux). When the central difference scheme is used to discretize these terms, artificial dissipations or TVD limiters are often used to suppress the numerical oscillations.

Other solution methods for the Navier-Stokes equations can also be used for the 2-D shallow water Eqs.(5)-(7). For example, the Rhie and Chow's (1983) momentum interpolation technique on non-staggered grid can also be used (Wu, 2004). Jia and Wang (1999) adopt the correction-type method to solve Eqs. (5)-(7).

4.3.3 Three-Dimensional Flows

In the full 3-D Navier-Stokes Eqs. (12) and (13), the momentum equations link the velocity to the pressure gradient, but the continuity equation does not directly link to the pressure and is just an additional constraint on the velocity field. Owing to such a weak linkage, the convergence and stability of a numerical solution of the Navier-Stokes equations depend largely on how the pressure gradient and velocity in these equations are evaluated. Storing the variables at the geometric center of the control volume coupled with a linear interpolation for internodal variation usually leads to non-physical node-to-node oscillation. One approach to avoid this numerical oscillation is to use the staggered grid adopted in the MAC method, the projection method and the SIMPLE algorithm. Another approach is to use the momentum interpolation technique proposed by Rhie and Chow (1983) based on the collocated (non-staggered) grid.

In 3-D simulations of river flow, the computational domain is movable due to the free-surface and bed-surface variation. One approach to handle this problem is ignoring the free-surface variation and treating the free-surface as a rigid lid boundary. The location of the rigid lid can be estimated by a 1-D or 2-D model. This approach is simple but only applicable to a short reach where the water surface elevation varies gently. Another approach is the volume-tracking method, which uses a fixed grid, and defines the shape and elevation of the water surface through the volume of fluid at each grid cell. The MAC and VOF algorithms (Hirt and Nichols, 1981) are examples of this group. The third approach is the surface-tracking method, which uses the moving grid that follows the free-surface change. At least one grid line is along the free surface so that the surface shape is approximately matched. In the surface-tracking method, the location of water surface is usually determined by the free-surface kinematic condition, the depth-integrated continuity equation, or the 2-D Poisson equation derived by Wu *et al*. (2000) from the 2-D depth-averaged momentum equations.

In the 3-D shallow water Eqs. (14)-(16), the pressure (water level) field becomes a 2-D quantity, but the weak linkage between the pressure and velocity still exists. Therefore, most of the current 3-D shallow water flow models adopt the staggered grid. Taking advantage of the 2-D feature of pressure, Sheng (1983) suggested the splitting of internal module and external module. The external module is the depth-averaged 2-D model, which handles the fast barotropic dynamics and computes the water level field. The internal module handles the slower baroclinic vertical flow structure by solving the 3-D equations. Casulli and Cheng (1992) proposed an algorithm that uses an implicit scheme in the vertical direction and a semi-implicit scheme in the horizontal direction. They derived a discrete 2-D Poisson equation to determine the water level.

4.4 Numerical Solutions of Sediment Transport

Although the physical interactions between the water and sediment always exist, the majority of the early sediment transport models have decoupled the flow and sediment calculations. Recently, intensive efforts have been made toward the relaxation of the limit in time and space steps and the extension of the applicability of the fully decoupled model. Recent research has found out that the coupled model is more stable (Holly and Rahuel, 1990; Cao *et al*., 2002).

However, the nonlinearity of flow problem may reduce the efficiency of solving the flow and sediment transport equations simultaneously. In addition, the sediment in the vast majority of rivers has very low concentration, and the time scales of flow and channel morphodynamic processes can be significantly different, especially in the case where bed load is dominant. Therefore, fully coupling the flow and sediment transport are usually not cost-effective. Wu and Vieira (2002) adopted a "semi-coupling" procedure, in which the flow calculation is decoupled from sediment calculation, but the three components of the sediment module (sediment transport, bed change and bed material sorting) are solved in a coupled fashion. This semi-coupling procedure has been found to be very stable and efficient computationally.

5 DOMAIN DECOMPOSITION AND MODEL INTEGRATION

Many problems in river engineering exhibit multiple length and time scales in the form of velocity, concentration, geometric irregularities and complexities. To handle these characteristics, domain decomposition and model integration are usually adopted to allow models having different dimensions, different physical processes, and different grid shapes and densities in different subdomains. These include the multiblock algorithm based on the domain decomposition, the coupling of 1-D, 2-D and 3-D models, the integration of channel and watershed models, and the hybrid physical and computational modeling.

5.1 Multiblock Method

The multiblock method divides the solution domain into several subdomains (blocks), and generates a structured mesh for each individual subdomain independently. The governing equations are solved bock by block. During the solution process, the information updated at each time step or iteration step needs to be transferred between the blocks. Therefore, the key issues in the multiblock method are the interface treatment and information exchange between blocks, which affect the solution accuracy and computation efficiency. For a complex problem, the multiblock method needs less computer memory and has more flexibility of grid generation than the single-block method. The multiblock algorithm is often used with parallel computation.

5.2 Coupling of 1-D, 2-D and 3-D Models

In the investigation of River Engineering problems, such as navigation, water intaking, river restoration, structure protection, etc., one needs to know the detailed flow, sediment transport and channel evolution, which should be simulated using a 2-D model or 3-D model. However, if a long river reach is studied over a long time period, it may be more cost-effective to use an integrated 1-D, 2-D and/or 3-D model, e.g. McAnally *et al*. (1986) and Wu and Li (1992). The basic idea is dividing the whole study domain into several subdomains (reaches), and applying the 1-D model in less important subdomains with simple geometry and the 2-D model and/or 3-D model in more important subdomains with complex geometry. The concept of the coupling of 1-D, 2-D and 3-D models is illustrated in Fig. 2 for a generic river system.

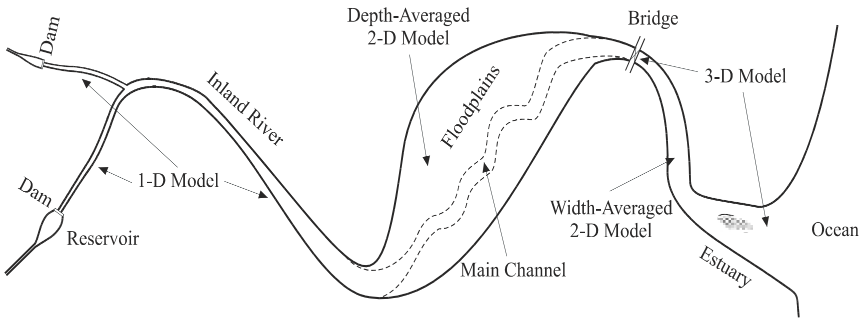


Fig. 2 Concept of Coupled 1-D, 2-D and 2-D Models

The solution schemes in each subdomain are the same as those discussed previously. On the interfaces between subdomains the flow flux, momentum and energy as well as sediment flux, bed change and bed material gradation should be conserved. The maximum computation time steps allowed by numerical stability in 1-D, 2-D and 3-D models are usually different. Therefore, the overall time step should be carefully selected.

5.3 Integration of Channel and Watershed Models

In traditional approach, the watershed and channel processes have been simulated separately. However, because they are interrelated, watershed and channel modeling should be integrated for a successful watershed-scale study. In the integrated system, the two parts can complement each other to produce better predictions.

As an example, the integration of the channel network model CCHE1D and the watershed models AGNPS and SWAT (Wu and Vieira, 2002) is used to illustrate the concepts and methodologies. This integrated watershed-channel modeling system includes three components: landscape analysis, watershed modeling, and channel simulation, as shown Fig.3. The landscape analysis program is used to extract the channel network and its corresponding subcatchments based on the elevation data from a Digital Elevation Model. The watershed models compute daily runoff and sediment yield for each subcatchment. The channel model simulates the flow and sediment routing in the channel network using the boundary conditions provided by the watershed models.

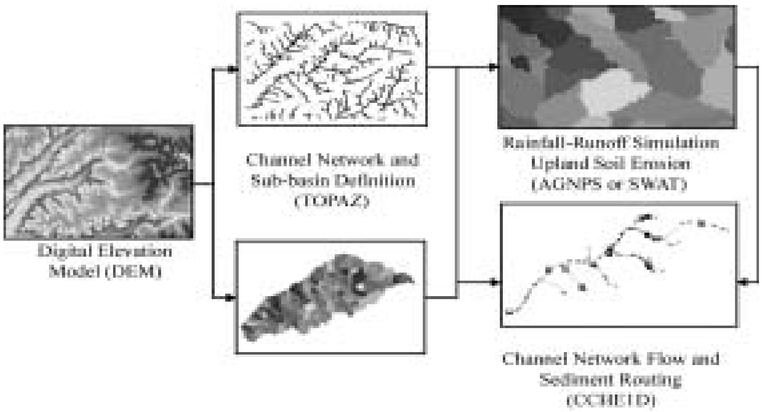


Fig. 3 Integration of Channel and Watershed Models

5.4 Hybrid Physical and Computational Modeling

As two major tools in river engineering analysis, both the physical (scale) modeling and computational modeling have advantages and disadvantages. The physical modeling is directly visible and measurable quantitatively, but more expensive and time-consuming. The computational modeling is less costly and requires less time to carry out. Due to this fact, the physical modeling and computational modeling are often combined in the investigation of very important engineering problems. This approach was called the hybrid physical and computational modelling (Mc Anally *et al.*, 1986).

Several ways have been adopted in the combination of both models. One typical approach is conducting physical model to study several scenarios for a real-life problem and collect measurement data to calibrate the computational model, and then extending the calibrated computational model to investigate more scenarios for this problem.

Another approach is using the computational model, especially the 1-D model or 2-D model, to investigate the effect of a hydraulic project in whole domain, and then using the physical model for a detailed study of local problems of critical importance. The computational model provides boundary conditions for the physical model. This approach has been adopted in the investigation of the sedimentation problems induced by the well-known Three Gorge Project in the Yangtze River.

6 MODEL VERIFICATION, VALIDATION AND APPLICATION

The common practice of applications of numerical models to the investigation of river sedimentation problems as well as other problems relies heavily on the calibration of model parameters using the measured data taken from the site under investigation. If the model being used has no major flaws, the researchers and engineers could usually have the values of the parameters adjusted or so-called calibrated so that the numerical model results can still approximately match the measurements. This has been claimed as validation of the numerical model. As a simple example, a river reach with highly irregular planform, varying cross-section shape, slope, bed and bank roughness, vegetation effect, etc. along the channel is modeled by assuming that the channel reach is a connected system of straight reaches (numerical elements) with uniform cross-section, slope, roughness, and vegetation effect. The users of the model could adjust the uniform bed roughness either along each element or even the entire reach to get the best approximate agreement between the modeled surface elevation profile and that measured. Once this is done the model is declared as validated. Quite often the so-called calibrated bed-roughness can be one order of magnitude higher than the one measured. This is due to the fact that the model user is using the bed roughness to include all the unrealistic assumptions of the model to obtain the agreement between simulated and measured water surface profiles. Even the model user can claim that the simulated water surface elevation along the river reach is validated by the measurements, one can not be sure that the discharge, velocity and sediment transport rate can be agreed also with their corresponding measurements.

More recently, the methodology of parametric identification has been advanced to the utilization of the multi-variable optimization methods. Its accuracy and reliability are still doubtful, especially due to the fact that the measured data are insufficient and incomplete in spatial and temporal resolutions. Even if one has a relatively large amount of data of a study site, the calibrated parameters may validate the model results of this specific site over the time period when the data was taken, one can not be sure that the results are valid at this site all the time, nor they are valid at a site outside the reach where the data was taken.

Furthermore, the approximate agreement between model results and measured data by calibrating the parameter does not guarantee that the model is correct mathematically. Therefore, it is of paramount importance to have all numerical models verified and validated comprehensively. A 3D Free Surface Flow Model Verification and Validation Monograph is to be published by a Task Committee of the same title chaired by Wang (2004). Its comprehensive procedure is briefly given below, which can also be applied to the development of sediment transport model. This verification and validation procedure consists of three steps.

<u>Step 1: Verification by Analytic Solutions:</u>

A numerical model may have problems in mathematics and physics. The verification and validation procedure developed by the ASCE Task Committee is to insure that a numerical model shall solve the mathematical equations correctly and predict the physical processes adequately and realistically. The verification of correct mathematics can be achieved by testing the model's results with analytic solutions. In this test, the numerical and analytical solutions are obtained from the same equations with same boundary and initial conditions and same values of all parameters. Therefore, the numerical solution should converge to the analytical solution in a grid refining process. Any discrepancy must have come from the mathematical derivation, numerical solution technique, computer coding and calculation algorithm. This step can not only prove consistency and convergence, but also determine the magnitude of model's numerical error quantitatively. Therefore, every model must be verified to have the correct mathematics first, before concerning how accurate it can predict the physical processes. Recent development in manufacturing nonlinear analytic solutions has greatly broadened the effectiveness of this verification step to test the nonlinear cases.

Step 2: Validation by Laboratory Experiments:

The current state of the technology and the constraint of economics have severely limited our ability to measure all forcings acting on a natural system and the responses of all the state variables of the system with sufficient spatial and temporal resolutions. Therefore, the validation of a numerical model's capability in prediction of basic physical mechanisms and realistic natural processes are best be carried out in two steps by using laboratory experiments and field measurements respectively.

The laboratory experiments are conducted in controlled environments, smaller in scale, less costly to measure more properties at finer resolution of the measuring points, and their simplified testing conditions are close to the numerical model than those of natural systems. Therefore, the comparison between the measured values with numerical solutions is more meaningful. Because of the beauty of science it has proven that the simplest physical processes are the fundamental modes of the physical system, so that the validation of this step can confirm the model's capability in predicting the most fundamental physical processes, though they are simplified. Another advantage of these tests is that there are no scaling distortion effects.

Step 3: Validation by Field Measurements:

Numerical models pass the first two steps may not be appreciated completely by practitioners. They need proof that the model has the capability of predicting the behavior of natural phenomena, at least approximately. Therefore, before the application of a model to the investigation of a real-life problem in nature, it has to be tested by field data. One needs to be aware, however, that the application of a model to each real-life problem has to have its parameters representing the unique characteristics of the study site determined by data measured at the site. This is called site specific calibration. In ASCE's monograph (to be published shortly), the Task Committee suggested that the user can not use all measured data in calibration. Instead at least half or more data should be kept for validation to confirm whether and how close or adequate the model can predict the natural phenomena.

One should understand, however, that the conditions considered by the numerical model have included several simplifying assumptions or omitted some complicated forcing of secondary importance. Therefore, one should not use this step to judge a model's accuracy quantitatively by comparing the simulation to the measurement at a point and a particular time. Normally, more measured field data available leads to more reliable model predictions. The most important contribution of this step is to predict the trend of spatial and temporal variations of a real-life system in nature correctly and realistically.

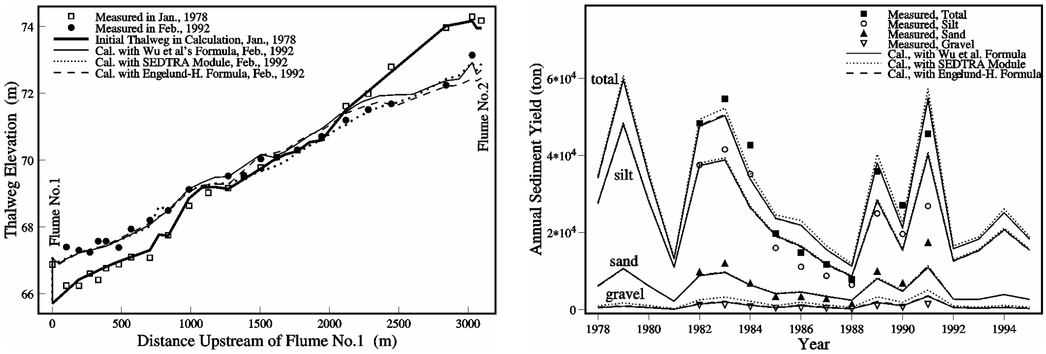
Because the sediment transport model has more physical parameters than the flow model, such as sediment transport capacity, non-equilibrium adaptation length, eddy diffusivity, etc., and they most likely vary in different river reaches in different seasons, the flow model should pass the above verification and validation tests first, and then, the sediment transport model should go through a similar procedure.

7 EXAMPLES OF APPLICATIONS

To better utilize the limit space of this paper, representative examples are selected to highlight the significant advances of the computational modeling in river engineering research rather than a comprehensive summary. Detailed information can be easily obtained from numerous publications including those cited in this paper.

7.1 Case 1: Integrated Watershed-Channel Simulation in Goodwin Creek

The Goodwin Creek watershed in Mississippi is the only watershed in the USA, which has been monitored continuously since 1978. Fig. 4(a) shows the comparison of the calculated and measured thalweg changes of the main channel from 1978 to 1992. Fig. 4(b) shows the comparison of the calculated and measured annual sediment yields at the watershed outlet. The reasonable agreement proves that computational model, CCHE1D, is capable of simulating long-term morphological changes of a stream.



(a) Thalweg Changes along the Main Stream; (b) Sediment Yields at Outlet

Fig. 4 Sediment Transport in Goodwin Creek

7.2 Case 2: Local Scour around Bridge Piers and Abutments

The results shown in Fig. 5 proves that CCHE3D model has been successfully applied to simulate the development of local scours around bridge pier and bridge abutment (Dou, 1997).

7.3 Case 3: Channel Widening and Meandering Simulation

Figs. 6 and 7 demonstrate the capabilities of computational models in simulating the river widening and meandering due to bank erosions, bed aggradation and degradation. Both CCHE2D and CCHE3D have been applied to this study.

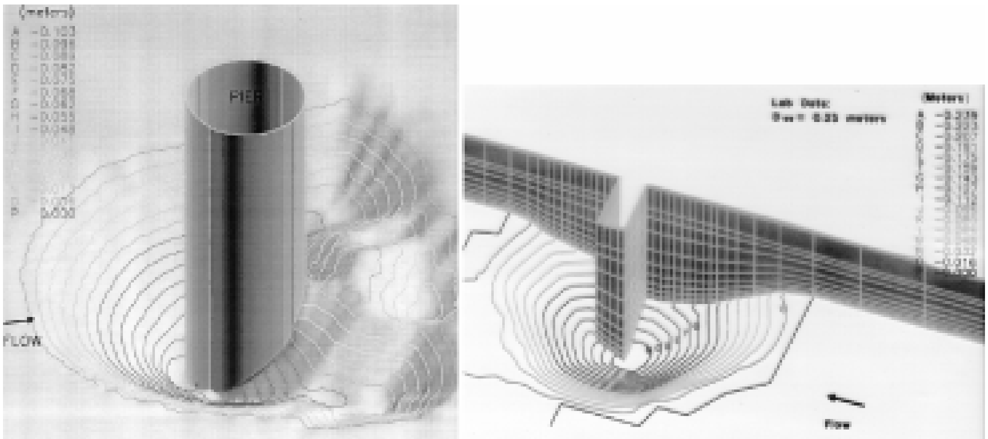


Fig. 5 Local Scours around a Bridge Pier and an Abutment

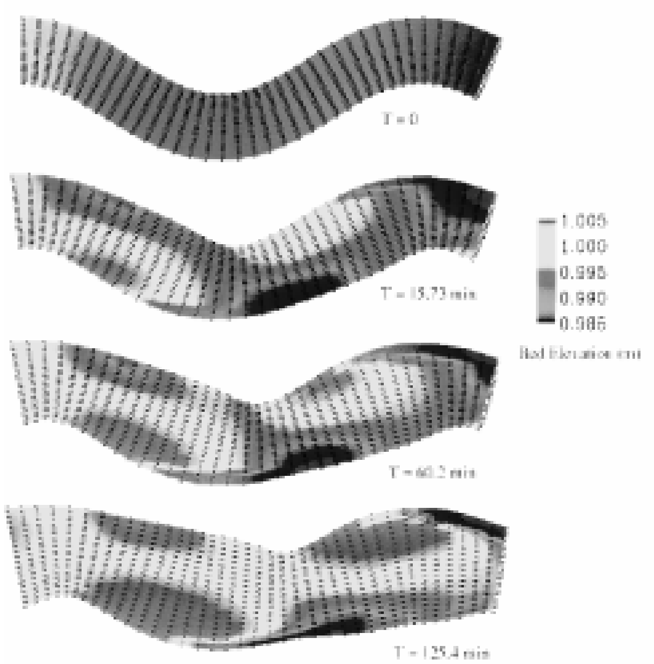


Fig. 6 Channel Widening

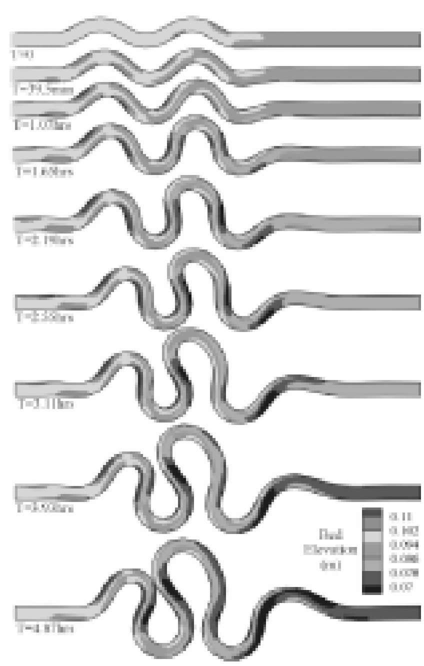


Fig. 7 Channel Meandering

8 CONCLUDING SUMMARY

River sedimentation and morphological modeling has been advanced significantly over the past two decades. Mandated by the U. S. Congress, the National Center for Computational Hydroscience and Engineering (NCCHE) has been developing the numerical-empirical models for simulating soil erosion and sediment transport since 1989. Not only a suite of basic simulation models in the areas of free-surface flow, sediment transport, channel bed/bank erosion, channel meandering, local scours, dam-break and flood flows, water quality, ecological impact, environmental engineering, etc., have been developed, but also the forefront of the state of the art of the modeling has been continuously advanced ever since. Advances of modeling methodology of NCCHE and other institutions have been summarized in the previous sections. At the present, the computational models have been widely adopted as alternative research, engineering analysis and design tools. Quite often, they have been the tools of choice by both researchers and practitioners, because of the constraints of available time and funds. From the examples presented in the last section, it is seen that the capabilities of computational simulation models have both broadened and strengthened to the extent beyond the belief of many experts in the field a few years ago.

At the present state of the art, the three-dimensional models can simulate the detailed flow characteristics of complex situation, e.g. the turbulent flow in a highly irregular river bendway with spur dikes and submerged weirs, the unsteady flows at bridge crossing and the local scour development in time around bridge piers and abutments, etc. The two-dimensional models have successfully applied to the predictions of flood flows due to over topping of river banks, dam break or levee breach caused by heavy rain storms, the selection of designs for river restoration projects by a variety of techniques including the utilization of vegetations and hydraulic structures in stream or along the banks, etc. The one-dimensional models are being used in assessing long-term sediment and/or pollutant transport in streams and channel networks, predicting the total maximum daily loads in a catchment or watershed, evaluating the effectiveness of erosion control structures from long-term point of view, and many other cases. The integrated multi-dimensional models having 1-D, 2-D and 3-D models are becoming the predictive tools in decision support systems, which are to have wider and wider applications in policy making, management planning and engineering designs to select the best management practices and the optimal design to satisfy all constraints of environment, ecology, political/legal systems, social/cultural concerns, etc. There is no doubt that the computational modeling is to be the preferred tool of the 21st century in River Engineering. The important areas for future developments in the river sedimentation modelling are recommended below:

One of the most important needs in sediment transport modeling is the betterment of understanding of sediment transport theory, especially the mechanism quantifying the dynamics of interactions between flowing water, sediment particles, and/or bed/bank materials. The advances in this area shall reduce the empiricism needed in the computational model, and thus enhance the models' completeness, effectiveness and efficiency.

The next urgent need is to intensify the laboratory experiments and field measurements on both mechanisms and natural phenomena of sediment transport processes in rivers. It is highly desirable to carry out these researches collaboratively with computational simulations, so that the large amount of data collected can be used by both physical and numerical modelers. Furthermore, the fully validated numerical models can be used to generate large amount of knowledge without having to run a large number of laboratory experiments and collect more field data than the amount absolutely necessary. The information collected can improve not only the accuracy and reliability of empirical functions that can be used immediately by the numerical modelers and practice engineers, but also by the academician to improve the theory and thus enhance our basic understanding.

Without any doubt, another future need in computational models for river sedimentation is to further upgrade the models. The newly proved sediment transport theories, the newly found empirical functions, the newly developed numerical solution methods, etc. should all be added to the computational models. These shall enhance the models' accuracy and reliability in predicting sediment transport capacity, sediment diffusivity, non-equilibrium adaptation length, movable bed roughness, near-bed sediment exchange mechanism, etc. Furthermore, the utilization of newly developed information technologies, such as GIS, GUI and software library, can greatly enhance the convenience in models' usage and broaden their applications, especially to the urgent real-life problems of our society.

With the advances of science and engineering in all fields, the solution of a problem in one discipline needs to consider its effects and impacts on other disciplines, from short- and long-term point view. It is also true that to solve a problem at one location, one must consider its effects not only on the neighboring area, but also on the entire watershed. The International Hydrological Program's International Sedimentation Initiative and the U. S. Army Corp of Engineers are respectively studying the need of consideration of global and regional impact when a sedimentation project is planned. Therefore, to design an engineering solution of a river sedimentation problem, one must integrate several models from other disciplines, such as ecosystem model, water quality model, pollutant transport model, even economic, management, social systems models, etc. The integration of all of these models together with all relevant knowledge and information databases into one comprehensive software package as a decision support system to be used by engineers, managers, administrators and policy makers. This decision support system shall be widely adopted in the future.

ACKNOWLEDGEMENTS

This study is part of the research project sponsored by the USDA-ARS Specific Research Agreement No.58-6408-2-0062 (monitored by the USDA-ARS National Sedimentation Laboratory) and the University of Mississippi.

REFERENCES

Ackers, P. and White, W.R., 1973, Sediment transport: A new approach and analysis, *J. Hydr. Div.*, ASCE, 99(HY11), 2041-2060.

Armanini, A. and Di Silvio, G., 1988, A one-dimensional model for the transport of a sediment mixture in non-equilibrium conditions, *J. Hydr. Res.*, 26(3).

Arulanandan, K., Gillogley, E. and Tully, R., 1980, Development of a quantitative method to predict critical shear stress and rate of erosion of naturally undisturbed cohesive soils, *Rep. GL*-80-5, U.S. Army Engineers Waterway Experiment Station, Vicksburg, Mississippi.

Bell, S. G. and Sutherland, A. J., 1983, Non-equilibrium bed load transport by steady flows, *J. Hydr. Engrg.*, 109(3), 353-367.

Brooks, N.H., 1963, Discussion of “boundary shear stresses in curved trapezoidal channels” by A.T. Ippen and P.A. Drinker, *J. Hydr. Div.*, ASCE, 89(HY3).

Brownlie, W.R., 1981, Compilation of fluvial channel data: laboratory and field, *Rep. No. KH-R-43B*, W.M. Keck Lab. of Hydr. and Water Resources, California Institute of Technology, Pasadena, Calif.

Cao, Z., Day, R. and Egashira, S., 2002, Coupled and decoupled numerical modeling of flow and morphological evolution in alluvial rivers, *J. Hydr. Engrg.*, ASCE, 128(3), 306-321.

Casulli, V. and Cheng, R.T., 1992, Semi-implicit finite difference methods for three-dimensional shallow water flow, *Int. J. for Numer. Methods in Fluids*, 15, 629-648.

Cunge, J. A., Holly, F. M. Jr. and Verwey, A., 1980, *Practical Aspects of Computational River Hydraulics*. Pitman Publishing Inc., Boston, MA.

De Vriend, H.J., 1981, Steady flow in shallow channel bends, *Communications on Hydraulics,* 81-3, Delft University of Technology, Delft, The Netherlands.

Dou, G.R., 1963, Sediment Transport Theory, Nanjing Hydraulic Research Institute, China. (in Chinese)

Dou, X., 1997, Numerical simulation of three-dimensional flow field and local scour at bridge crossings, *PhD Dissertation*, The University of Mississippi, USA.

Duan, J.G, Wang, S.S.Y. and Jia, Y., 2001, The application of the enhanced CCHE2D model to study the alluvial channel migration processes, *J. Hydr. Res.*, IAHR, 39(5), 469-480.

Einstein, H.A. and Barbarossa N.L., 1952, River channel roughness, *Trans.* ASCE, 117, 1121-1132.

Einstein, H.A., 1950, The bed-load function for sediment transportation in open channel flows, *Technical Bulletin* No. 1026, U.S. Department of Agriculture, Soil Conservation Service, Washington D.C.

Engelund, F. and Hansen, E., 1967, *A Monograph on Sediment Transport in Alluvial Streams*, Teknisk Vorlag, Copenhagen, Denmark.

Engelund, F., 1974, Flow bed topography in channel bend, *J. Hydr. Div.*, ASCE, 100(11), 1631-1648.

FHWA, 1995, Evaluating scour at bridges, HEC-18, Third Edition, Federal Highway Administration, U.S. Department of Transportation.

Flokstra, C., 1977, The closure problem for depth-average two dimensional flow, *Publication* No. 190, Delft Hydraulics Laboratory, The Netherlands.

Garbrecht, J., Kuhnle, R. and Alonso, C., 1995, A sediment transport capacity formulation for application to large channel networks, *J. Soil and Water Conserv.*, 50(5), 527-529.

Garcia, M. and Parker, G., 1991, Entrainment of bed sediment into suspension, *J. Hydr. Engrg.*, ASCE, 117(4), 414-433.

Gibson, R.E., England, G.L. and Hussey, M.J., 1967, The theory of one-dimensional consolidation of saturated clays, *Geotechnique*, 17, 216-263.

Hirt, C.W. and Nichols, B.D., 1981, Volume of fluid (VOF) method for dynamics of free boundaries, *J. Computational Physics*, 39, 201-221.

Holly, F.M., Jr. and Rahuel, J. L., 1990, New numerical/physical framework for mobile-bed modeling, Part I: Numerical and physical principles, *J. Hydr. Res.*, IAHR, 28(4), 401-416.

Jia, Y. and Wang, S.S.Y., 1999, Numerical model for channel flow and morphological change studies, *J. Hydr. Engrg.*, ASCE, 125(9), 924-933.

Jia, Y., Kitamura, T. and Wang, S.S.Y., 2001, Simulation of scour process in plunging pool of loose bed-material, *J. Hydr. Engrg.*, ASCE, 127(3), 219-229.

Jin, Y.C. and Steffler, P.M., 1993, Predicting flow in curved open channels by depth-averaged method, *J. Hydr. Engrg.*, ASCE, 119(1), 109-124.

Karim, F. and Kennedy, J.F., 1982, IALLUVIAL: A computer-based flow- and sediment- routing model for alluvial streams and its application to the Missouri river, *Technical Report* No. 250, Iowa Institute of Hydraulic Research, University of Iowa, USA.

Krone, R.B., 1962, Flume studies on the transport of sediment in estuarine shoaling processes, Hydraulic Engineering Laboratory, University of Berkeley, California, USA.

Lane, E.W. and Koelzer, V.A., 1953, Density of sediments deposited in reservoirs, *Report* No. 9 *of a Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams*, Engineering District, St. Paul, MN, USA,

Leonard, B.P., 1979, A stable and accurate convective modelling procedure based on quadratic interpolation, *Comput. Meths. Appl. Mech. Eng*., Vol. 19.

Lopez, F. and Garcia, M., 2001, Mean flow and turbulence structure of open-channel flow through non-emergent vegetation, *J. Hydr. Engrg.*, ASCE, 127(5), 392-402.

McAnally, W.H., Letter, J.V. and Thomas, W.A., 1986, Tow and three-dimensional modeling systems for sedimentation, *The Third Int. Symp. on River Sedimentation*, The University of Mississippi, USA, pp. 400-411.

Mehta, A.J. and Partheniades, E., 1975, An investigation of the depositional properties of flocculated fine sediment, *J. Hydr. Res.*, 13(4), 361-381.

Mehta, A.J., 1986, Charaterization of cohesive sediment properties and transport processes in estuaries, *Estuarine Cohesive Sediment Dynamics*, A.J. Mehta (ed.), Springer-Verlag, 290-325.

Meyer-Peter, E. and Mueller, R., 1948, Formulas for bed-load transport, *Report on Second Meeting of IAHR*, Stockholm, Sweden, 39-64.

Odgaard, A.J., 1981, Transverse bed slope in alluvial channels bends. *J. Hydr. Div.*, ASCE, 107(12), 1677-1694.

Osman, A.M. and Thorne, C.R., 1988, Riverbank stability analysis, I: Theory, *J. Hydr. Engrg.*, ASCE, 114(2), 134-150.

Parker, G., Kilingeman, P.C. and McLean, D.G., 1982, Bed load and size distribution in paved gravel-bed streams, *J. Hydr. Div.*, ASCE, 108(4), 544-571.

Partheniades, E., 1965, Erosion and deposition of cohesive soils, *J. Hydr. Divi.*, ASCE, 91(HY1).

Proffit, G.T. and Sutherland, A.J., 1983, Transport of nonuniform sediment, *J. Hydr. Res.*, IAHR, 21(1), 33-43.

Rahuel, J.L. and Holly, F.M., *et al*., 1989, Modeling of riverbed evolution for bedload sediment mixtures, *J. Hydr. Engrg.*, ASCE, 115(11), 1521-1542.

Raudkivi, A.J. and Hutchison, D.L., 1974, Erosion of Kaolinite Clay by flowing water, *Proceedings*, Royal Society, London, England, Series A, 337, 537-544.

Rhie, C.M. and Chow, W.L., 1983, Numerical study of the turbulent flow past an airfoil with trailing edge separation, *AIAA J.*, 21, 1525-1532.

Ribberink, J. S., Blom, A. and van der Sheer, P., 2002, Multi-fraction techniques for sediment transport and morphological modeling in sand-gravel rivers, *River Flow* 2002, Bousmar and Zech (eds.), Swets & Zeitinger, Lisse, 731-739.

Rodi, W., 1993, *Turbulence Models and Their Application in Hydraulics*, 3rd Ed., IAHR Monograph, Balkema, Rotterdam.

Rozovskii, I.L., 1957, *Flow of water in bends of open channel*, Academy of Sciences of the Ukrainian SSR, Kiev.

Sekine, M. and Parker, G., 1992, Bed-load transport on transverse slope, I, *J. Hydr. Engrg.*, ASCE, 118(4), 513-535.

Sheng, Y.P., 1983, Mathematical modeling of three-dimensional coastal currents and sediment dispersion: Model development and application, *Technical Report* CERC-83-2, Aeronautical Research Associates of Princeton, Inc., N.J.

Shimizu, Y. and Tsujimoto, T., 1994, Numerical analysis of turbulent open-channel flow over a vegetation layer using a k-ε turbulence model, *J. of Hydroscience and Hydraulic Engrg.*, JSCE, 11(2), 57-67.

Simon, A., Curini, A., Darby, S.E. and Langendoen, E.J., 2000, Bank and near-bank processes in an incised channel, *Geomorphology*, 35, 193-217.

Spalding, D.B., 1972, A novel finite-difference formulation for differential expressions involving both first and second derivatives, *Int. J. Num. Meth. Eng.*, Vol. 4.

Spasojevic, M. and Holly, F.M. Jr., 1993, Three-dimensional numerical simulation of mobile-bed hydrodynamics, *Technical Report* No. 367, Iowa Institute of Hydraulic Research, The University of Iowa, USA.

Stone, H.L., 1968, Iterative solution of implicit approximation of multidimensional partial differential equations, *SIAM J. on Numerical Analysis*, 5, 530-558.

Thomas, W.A., 1982, Chapter 18: Mathematical modeling of sediment movement, *Gravel-bed Rivers*, R.D. Hey, et al., eds., John Wiley & Sons, Ltd., New York, N. Y.

Thorn, M.F.C., 1981, Physical processes of siltation in tidal channels, *Proceedings, Hydraulic Modelling Applied to Maritime Engineering Problems*, ICE, London, 47-55.

Van Rijn, L.C., 1989, Handbook: sediment transport by current and waves, *Report H 461*, Delft Hydraulics.

Wang, S.S.-Y. and Adeff, S.E., 1986, Three-dimensional modelling of river sedimentation processes, *Proc. of the 3rd Int. Symp. on River Sedimentation*, The Univ. of Mississipi, USA.

Wang, S.S.Y. and Hu, K.K., 1992, Improved methodology for formatting finite element hydrodynamic models, *Finite Element in Fluids*, Ed. by T.J. Chung, Vol. 8, Hemisphere Publishing Corp., 457-478.

Wang, S.S.Y. et al., 2004, *3D Free-Surface Flow Model Verification and Validation*, an ASCE Monograph to be published in Dec. 2004.

Wu, W. and Li, Y., 1992, One- and two-dimensional nesting mathematical model for river flow and sedimentation, *The Fifth Int. Symp. on River Sedimentation*, Karlsruhe, Germany, 1, 547-554.

Wu, W. and Vieira, D.A., 2002, One-dimensional channel network model CCHE1D 3.0 -- Technical manual, *Technical Report No. NCCHE-TR-2002-1*, National Center for Computational Hydroscience and Engineering, The University of Mississippi.

Wu, W. and Wang, S. S.Y., 1999, Movable bed roughness in alluvial rivers, *J. Hydr. Engrg.*, ASCE, 125(12), 1309-1312.

Wu, W. and Wang, S. S.Y., 2004a, Depth-averaged numerical modeling of flow and sediment transport in open channels with vegetation, *Riparian Vegetation and Fluvial Geomorphology: Hydraulic, Hydrologic and Geotechnical Interactions*, S.J. Bennett and A. Simon (eds.), American Geophysics Union. (in printing)

Wu, W. and Wang, S. S.Y., 2004b, Depth-averaged 2-D calculation of tidal flow, salinity and cohesive sediment transport in estuaries, Under review by *Int. J. of Sediment Research*.

Wu, W. and Wang, S. S.Y., 2004c, Empirical-numerical analysis of headcut migration, *The 3rd Int. Conf. on Gully Erosion*, April 28-May 1, Oxford, MS, USA.

Wu, W. and Wang, S. S.Y., 2004d, Depth-averaged 2-D calculation of flow and sediment transport in curved channels, Under review by *Int. J. of Sediment Research*.

Wu, W. and Wang, S.S.Y., 2003, Selection and evaluation of nonuniform sediment transport formulas for river modeling, *The XXX IAHR Congress,* Thessaloniki, Greece.

Wu, W., 2004, Depth-averaged 2-D numerical modeling of unsteady flow and nonuniform sediment transport in open channels, *Under Review by J. of Hydraulic Engineering*, ASCE.

Wu, W., Rodi, W. and Wenka, T., 2000, 3-D numerical modeling of water flow and sediment transport in open channels, *J. Hydr. Engrg.*, ASCE, 126(1), 4-15.

Wu, W., Wang, S.S.-Y. and Jia, Y., 2000, Nonuniform Sediment Transport in Alluvial Rivers, *J. Hydr. Res.*, IAHR, 38(6), 427-434.

Yalin, M.S., 1972, *Mechanics of Sediment Transport*, Pergamon Press.

Yang, C.T. and Wan, S., 1991, Comparisons of selected bed-material load formulas, *J. Hydr. Engrg.*, ASCE, 117(8), 973-989.

Yang, C.T., 1973, Incipient motion and sediment transport, *J. Hydr. Div.*, ASCE, 99(HY10), 1679-1704.

Ying, X., Khan, A.A. and Wang, S.S.Y., 2003, An upwind method for one-dimensional dam break flows, *Proceeding of XXX IAHR Congress*, Greece, pp. 245-252.

Zhang, R.J. and Xie, J.H., 1993, *Sedimentation Research in China, Systematic Selections*, China Water and Power Press, Beijing.

Zyserman, J.A. and Fredsϕe, J., 1994, Data analysis of bed concentration of suspended sediment, *J. Hydr. Engrg.*, ASCE, 120(9), 1021-1042.